FRACSIMA

Fracture Simulation in Materials
4 Impasse Doron – 94110 – Arcueil – France
www.fracsima.com

January 18, 2026

Hydraulic diffusion in unsaturated material

Example of User module to model unsaturated flow

The flow in the matrix is governed by the Darcy's law:

$$\underline{v} = -\frac{\boldsymbol{k}^{in}}{\mu} k_r(S)\, \nabla(p + \gamma_w z) \tag{1}$$

with $\underline{v}$ the fluid velocity, p the pressure, μ the dynamic viscosity of the fluid, $\boldsymbol{k}^{in}$ the intrinsic permeability tensor and k_r the relative permeability function of the *saturation degree* S :

$$k_r(S) = \sqrt{S}\left(1-(1-S^{1/m'})^{m'}\right)^2 \tag{2}$$

m' is a material constant (See also [1]). γ_w is the water pressure gradient given in the general parameters of the model (Problem Type).

The mass balance equation in the matrix reads:

$$div(\rho_f\, \underline{v}) + \partial m_f / \partial t = 0 \tag{3}$$

where m_f is the fluid mass present in the unit volume of the matrix and ρ_f the fluid specific mass. In unsaturated conditions:

$$m_f = \rho_f\, \phi\, S \tag{4}$$

with ϕ the porosity and S saturation degree.

The variation of ϕ is given by the poromechanical model of the matrix and is related. But here we consider a pure hydraulic model and therefore *assume the porosity constant,* so $d\phi=0$. For an extension to poroelastic behavior the equation $d\phi = a_m d\varepsilon_v + a_h dp$ from the paper [2] can be used.

To make possible a standard formulation of the the governing equations for numerical modelling by FEM, the variation of m_f is to be related to that of the nodal variable p. Starting with:

$$\frac{dm_f}{m_f} = \frac{d\rho_f}{\rho_f} + \frac{dS}{S} \tag{5}$$

The variation of ρ_f is related to that of p by:

$$\frac{d\rho_f}{\rho_f} = \frac{dp}{K_f} \tag{6}$$

where K_f is the fluid compressibility.

For the saturated material, S=1 and the pressure p takes positive values. For unsaturated material, S<1 and its variation is function of the *suction s* as given by the *retention curve*. The suction s is the difference between the gas (air, vapor) pressure p_g and the fluid pressure p in the pore space: $s = p_g - p$. In UNSAT material, only two phases, solid skeleton and pore fluid are modeled, and then, if a gas phase exists, it is considered to be at constant zero pressure. This means that, in unsaturated state, $s = -p$. By extending the pressure values to the negative domain it is possible to represent with a unique variable p the pressure in saturated conditions ($p \geq 0$) and the suction in unsaturated conditions ($p<0$). The saturation degree is related to this variable by a function $S = f(s) = f(-p)$ coinciding with the retention curve in unsaturated conditions ($p < 0$) and extended by $S=1$ for $p \geq 0$ (see the Figure 1).

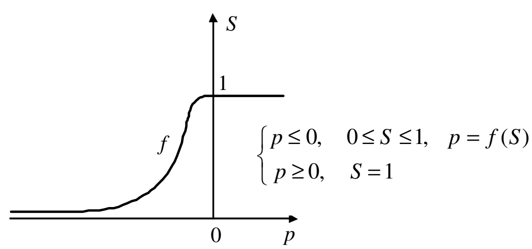

Figure 1. The retention curve f of the porous material extended to the saturated phase with $p > 0$, $S=1$

The time derivatives of S and p are then related by:

$$\frac{\partial S}{\partial t} = S' \frac{\partial p}{\partial t} \tag{7}$$

where:

$$\begin{cases} S' = 0 & \text{if} \quad p \geq 0, \quad S = 1 \\ S' = -\dfrac{df}{ds} & \text{if} \quad p \leq 0, \quad 0 \leq S \leq 1 \end{cases} \tag{8}$$

If, for instance, the Van Genuchten [3] law is chosen for the retention curve:

$$S = \frac{1}{\left(1 + (\alpha s)^n\right)^m} \tag{9}$$

with $\alpha > 0$, $0 \leq m < 1$ and $n = (1-m)^{-1}$. Then, for $p < 0$:

$$S' = \frac{-mnS}{p}\left(1 - S^{(1/m)}\right) \tag{10}$$

Replacing by the equations (1),(5),(6) and (7) in (3), the following governing equation is obtained for the pressure field evolution:

$$div(\boldsymbol{K}\nabla p) = C\frac{\partial p}{\partial t} \tag{11}$$

with:

$$\boldsymbol{K} = \frac{\rho_f}{\mu} \boldsymbol{k}^{in} k^r, \qquad C = \rho_f \phi \left(\frac{S}{K_f} + S' \right) \tag{12}$$

The assumption of incompressible fluid ($\rho_f = cste.$) brings further simplification. We can simply ρ_f from the two sides and write :

$$\boldsymbol{K} = \frac{\boldsymbol{k}^{in}}{\mu} k^r, \qquad C = \phi \left(\frac{S}{K_f} + S' \right) \tag{13}$$

The parameters to be specified for the materials are thus $\boldsymbol{k} = \boldsymbol{k}^{in}/\mu$, ϕ, K_f and the parameters m,n, α for the Van Genuchten law as well as the parameter m' for the relative permeability law. The water pressure gradient γ_w is given in the general parameters (Problem Type).

Nb = 7
Param1 = $k = k^{in}/\mu$ (permeability)
Param2 = ϕ (porosity)
Param3 = K_f (water compressibility)
Param4 = m' (for relative permeability)
Param5 = m (for Van Genuchten)
Param6 = n (for Van Genuchten)
Param7 = α (for Van Genuchten)

Internal variable:
$V^{in}_h(n,1)$: internal variable 1-S

Some examples of parameters values for Van Genuchten model can be found in [4].

References

[1] Disroc Materials Catalogue, model Gelisol 32111 (http://www.fracsima.com/DISROC/Materials-Catalog.pdf)

[2] Pouya A., A finite element method for modeling coupled flow and deformation in porous fractured media. Int. J. Numer. Anal. Meth. Geomech. 2015; 39:1836–1852, DOI: 10.1002/nag.2384

[3] Van Genuchten, M. T. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Science Society of America Journal, 1980; 44 (5): 892-898.

[4] Giulia Bella (2021). ‘Water retention behavior of tailings in unsaturated conditions’, Geomechanics and Engineering 26(2):15