FRACSIMA

Fracture Simulation in Materials

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DISROC

Materials' Catalogue

Finite Element Code Enriched with Joint Element for
Thermo-Hydro-Mechanical processes in Fractures Porous Media

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General Notation

For each material type, the code is composed of 5 digits:

The first digit is 1, 2,3, 4 for designating the elements nature:

1 for bar elements,
2 for joint elements (interfaces, cracks and fractures),
3 for surface elements (bulk materials),
4 for anchor elements.
5 for beam elements
6 for bolt elements.

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This second digit is 1 or 2 to designate the phenomena which is concerned:

1 for Mechanics,
2 for Hydraulics,
3 for Thermal.

The other 3 digits define the constitutive model.

For each material constitutive model, first the number of parameters, Nb, and then the values of the Nb parameters are specified.

I) Mechanics

I.1) Mechanics - BARS

11100 : Linear elastic bar element

Constitutive Relation : $F = E_s\, \varepsilon$
F : axial force
ε : axial deformation

Note: In 2D plane modeling, a unit the thickness of the model is considered in relation with a 3D modeling. The section S considered for the bar is so related to a unit thickness of the model. If, in the direction perpendicular to the plane of modeling, the bars are distant, for instance, of 40 *cm* and if the adopted unit length is meter, then there are 2.5 bars per unit thickness of the model. Then, the physical section of the bars has to be multiplied by this factor 2.5 to define S in the above relation, and then multiplied by the Young's modulus to define the above parameter E_s.
Example: The length unity is meter and the stress unity, *MPa*, and so the force unity, *MN* (Mega Newoton). Bars diameter is 2 *cm* and bars distance in the direction perpendicular to the plane of modeling equal to 40*cm*, and the steel Young's modulus 210.10^3 *MPa*. Then $E_s = \pi\times(0.01)^2 \times 2.5 \times 210.10^3 = 1650$ *MN*. The axial force calculated by the code is expressed in *MN* unity.

Nb = 1
Param1 = E_s (The product $E\times S$ of the Young modulus and the section of the bar.
Dimension : force)

11100	ELinear Elastic bar element
Nb: 1	
Param1 = E_s	

11110 : Linear elastic-plastic bar element

Constitutive Relation : $F = E_s\,(\varepsilon - \varepsilon^p)$
$d\varepsilon^p = 0$ if $\sigma < \sigma_y$ or if $\sigma = \sigma_y$ and $d\sigma < 0$
F : axial force
ε : axial deformation
ε^p : axial plastic deformation

Note: The section S for E_s and Y_s takes into account in the same way the bars distance in the direction perpendicular to the plane of the model: see the note for the material 11100. In every configuration, we must have $Y_s/E_s=\sigma_y/E$.

Nb = 2
Param1 = E_s (Product $E\times S$ of the Young modulus and the section of the bar. Dimension : force)
Param2 = Y_s (Product $\sigma_y S$, limit elastic force)

11110	Elastoplastic bar element
Nb: 2	
Param1 = E_s	
Param2 = Y_2	

I.2) Mechanics - ROCKJOINTS & FRACTURES

21100 : Linear elastic joint

Constitutive Relation: $\underline{\sigma} = \boldsymbol{K}\, \underline{u}$,

$$\begin{pmatrix} \tau \\ \sigma_n \end{pmatrix} = \begin{bmatrix} K_t & K_{tn} \\ K_{nt} & K_n \end{bmatrix} \begin{pmatrix} u_t \\ u_n \end{pmatrix}$$

Nb = 3
Param1 = K_t (tangent stiffness)
Param2 = K_n (normal stiffness)
Param3 = $K_{nt} = K_{tn}$ (non diagonal stiffness term, defining dilatancy)

Note: The stiffness parameters K_t, K_n, K_{tn} have the dimension of stress/length. Their values are highly depending on the physical properties of the fractures walls (roughness..) and/or of filling materials (for rockjoints). If a rockjoint is assimilated to a thin layer of thickness e of an elastic material with Young's modulus E and shear modulus μ, then $K_t = \mu/e$, $K_n = E/e$ and K_{tn}=0.

21100	Linear Elastic joint
Nb: 3	
Param1 = K_t (tangent stiffness)	
Param2 = K_n (normal stiffness)	
Param3 = $K_{nt} = K_{tn}$ (non diagonal stiffness → dilatancy)	

21120 : Linear elastic with Mohr-Coulomb plasticity

$$\underline{\sigma} = \boldsymbol{K}\,(\underline{u} - \underline{u}^p)$$

Elasticity: The model 21100
Plasticity : Mohr-Coulomb criterion:

$$f(\underline{\sigma}) = |\tau| + \sigma_n \tan\phi - c \leq 0$$

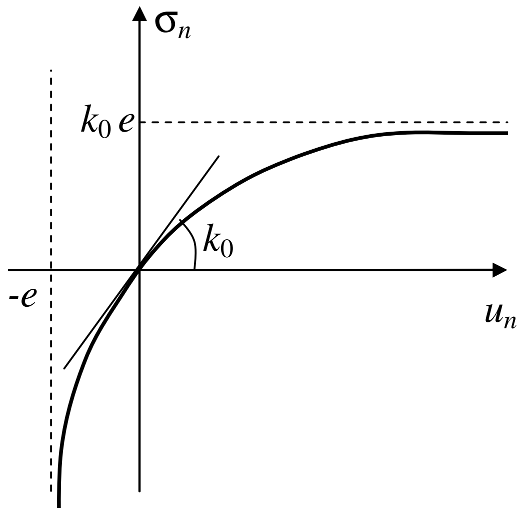
Nb = 5
Param1 = K_t
Param2 = K_n
Param3 = $K_{nt} = K_{tn}$
Param4 = c (cohesion)
Param5 = ϕ (in degrees, the friction angle)

21120	Linear Elastic joint with Mohr-Coulomb Plasticity
Nb: 5	
Param1 = K_t	
Param2 = K_n	
Param3 = $K_{nt} = K_{tn}$	
Param4 = c (cohesion)	
Param5 = ϕ (in degrees, the friction angle)	

21200 : Non-linear hyperbolic elasticity

The closure displacement is limited by the initial thickness e of the interface. The stress tends to infinity when closure displacement u_n tends to $-e$ and to $k_0 e$ for great positive openings:

$$\sigma_n = \frac{k_0 u_n}{1 + u_n / e}$$



The tangent behavior is linear:

$$\begin{cases} \sigma_t = k_t u_t + k_{nt} u_n \\ \sigma_n = k_{nt} u_t + \dfrac{k_0 u_n}{1 + u_n / e} \end{cases}$$

The normal stiffness k_n is u_n-dependant and given by:

$$k_n = \frac{k_0}{1 + u_n / e}$$

Nb = 4
Param1 = K_t (tangent stiffness)
Param2 = k_0 (normal stiffness)
Param3 = $K_{nt} = K_{tn}$ (non diagonal stiffness term causing dilatancy)
Param4 = e (maximum closure or physical thickness of the interface)

21200 Non-linear hyperbolic elastic joint

Nb: 4
Param1 = K_t (tangent stiffness)
Param2 = k_0 (normal stiffness)
Param3 = $K_{nt} = K_{tn}$ (non diagonal stiffness → dilatancy)
Param4 = e (maximum closure or physical thickness

21220 : Non-linear elasticity with Mohr-Coulomb plasticity

$$\underline{\sigma} = \boldsymbol{K}\,(\underline{u} - \underline{u}^p)$$

Elasticity: The model 21200
Plasticity : Mohr-Coulomb criterion:

Nb = 6
Param1 = K_t
Param2 = k_0
Param3 = $K_{nt} = K_{tn}$
Param4 = e
Param5 = c
Param6 = ϕ (degrees)

21220	Non-Linear Elastic joint with Mohr-Coulomb Plasticity
Nb: 6	
Param1 = K_t	
Param2 = k_0	
Param3 = $K_{nt} = K_{tn}$	
Param4 = e	
Param5 = c	
Param6 = ϕ (degrees)	

*

21230 : Non-linear elasticity with Lemaitre creep law

$$\dot{\underline{u}} = \dot{\underline{u}}^e + \dot{\underline{u}}^v$$

$$\underline{\sigma} = \boldsymbol{K}\,(\underline{u} - \underline{u}^v), \qquad \begin{pmatrix} \tau \\ \sigma_n \end{pmatrix} = \begin{bmatrix} K_t & K_{tn} \\ K_{nt} & K_n \end{bmatrix} \begin{pmatrix} u_t - u_t^v \\ u_n - u_n^v \end{pmatrix}$$

$$K_n = \frac{k_0}{1 + u_n / e}$$

Elasticity: the same that 21200

Viscous strain: Lemaitre creep law for uniaxial creep under constant stress σ with a stress threshold σ_c :

$$\varepsilon^v(t) = a <\sigma - \sigma_c>^q t^\alpha$$

where:

$$< x > = 0 \quad if \quad x < 0$$
$$< x > = x \quad if \quad x \geq 0$$

The incremental creep law uses the internal variable $\xi = \varepsilon^{1/\alpha}$ and reads:

$$\xi = \varepsilon^{1/\alpha}\ , \qquad \dot{\xi} = \left(a < \sigma - \sigma_c >^q\right)^{1/\alpha}, \qquad \dot{\varepsilon} = \alpha\, \xi^{\alpha-1} \dot{\xi}$$

To avoid numerical problems near ξ=0, the law is completed by: $\dot{\varepsilon} = \alpha\, \xi_0^{\alpha-1} \dot{\xi}$ if $\varepsilon \leq \varepsilon_0$.

This law is adapted to the joint shear and normal creeps.

For the normal creep:

$$\dot{\xi}_n = s_n \left(b_n h < |\sigma_n| - \sigma_c >^q\right)^{1/\alpha}; \qquad \dot{u}_n^v = \alpha\, \xi_n^{\alpha-1} \dot{\xi}_n$$

where $s_n = \pm 1$ is the sign of σ_n and b_n a constant parameter. The normal creep must be limited in order to avoid the closure exceeding e, or u_n falling below $-e$. The elastic law takes already into account this constraint. The parameter h , $0 \leq h \leq 1$, is introduced in order to satisfy this condition.

For shear creep, it is supposed that, the normal compressive stress decreases the slip rate, similar to frictional effects, and so the criterions depends on the normal stress also with a 'friction angle' parameter ϕ. It is also supposed that a traction normal stress has no effect on the viscous slip. This leads to the following expressions:

$$\dot{\xi}_t = s_t \left(b_t \left\langle |\tau| - \left\langle -\sigma_n \right\rangle \tan\phi - \tau_c \right\rangle^q\right)^{1/\alpha} \quad ; \qquad \dot{u}_t^v = \alpha\, \xi_t^{\alpha-1} \dot{\xi}_t$$

where $s_t = \pm 1$ is the sign of τ and b_t a constant parameter

Nb = 12
Param1 = K_t (tangent stiffness)
Param2 = k_0 (normal stiffness)
Param3 = $K_{nt} = K_{tn}$ (non diagonal stiffness term causing dilatancy)
Param4 = e (maximum closure or physical thickness of the interface)
Param5 = q
Param6 = α
Param7 = b_t
Param8 = b_n

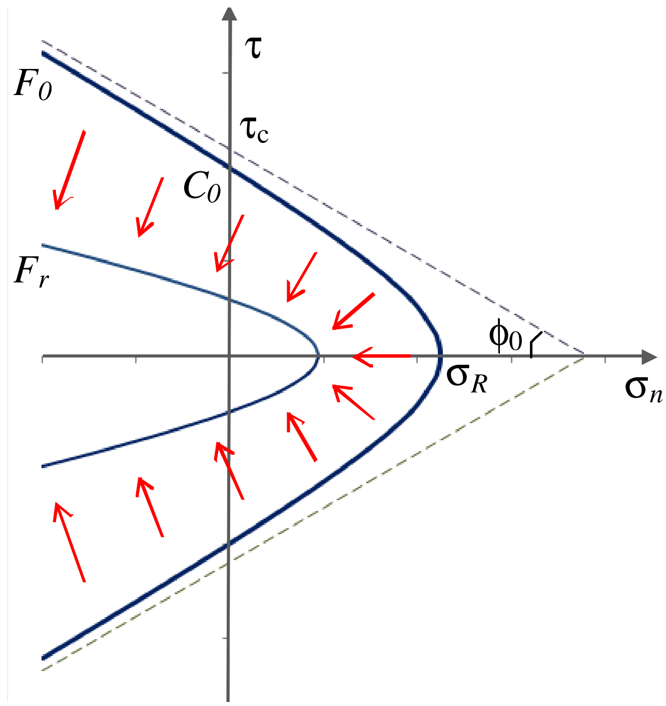
Param9 = τ_c
Param10 = σ_c
Param11 = ϕ (°)
Param12 = ε_0

Internal Variables:
Vin(n,1): internal for non linear elasticity
Vin(n,2) : not existing for this material
Vin(n,3) =ξ_t , Vin(n,4) =ξ_n

21230 Non-Linear Elastic joint with Lemaitre creep law

Nb: 12
Param1 = K_t
Param2 = k_0
Param3 = $K_{nt} = K_{tn}$
Param4 = e (maximum closure)
Param5 = q
Param6 = α
Param7 = b_t
Param8 = b_n
Param9 = τ_c
Param10 = σ_c
Param11 = ϕ (°)
Param12 = ε_0

21240 : Non-linear elastoplastic with Pouya strength criterion and with softening



Strain : $\underline{\dot{u}} = \underline{\dot{u}}^e + \underline{\dot{u}}^p$

Elasticity: the same that 21200

$\underline{\sigma} = \boldsymbol{K}\,(\underline{u} - \underline{u}^p)$,

$$\begin{pmatrix} \tau \\ \sigma_n \end{pmatrix} = \begin{bmatrix} K_t & K_{tn} \\ K_{nt} & K_n \end{bmatrix} \begin{pmatrix} u_t - u_t^p \\ u_n - u_n^p \end{pmatrix}$$

Non linear modulus:

$$K_n = \frac{k_0}{1 + u_n / e}$$

Strength criterion (plasticity): $$F(\underline{\sigma},\xi) = \sqrt{\tau^2 + b^2 g^2} + h\,\sigma_n \tan\phi_0 - g\,\tau_c$$

$$\tau_c = \frac{C_0^2 + \sigma_R^2 \tan^2\phi_0}{2\sigma_R \tan\phi_0}\ , \qquad b = \tau_c - \sigma_R \tan\phi_0$$

The initial strength function F_0 and the residual F_r are hyperbolic surfaces represented in the figure. The evolution from to the other results from the variation of the g and h which are evolution functions for the cohesion and friction angle and vary from 1 (initial state) to residual values respectively g_r and h_r:

$$g_r = \frac{\tau_r}{C_0} = \frac{\sigma_r}{\sigma_R}$$

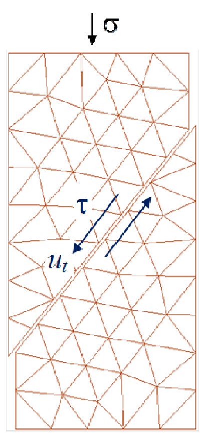
$$h_r = \frac{\phi_r}{\phi_0}$$

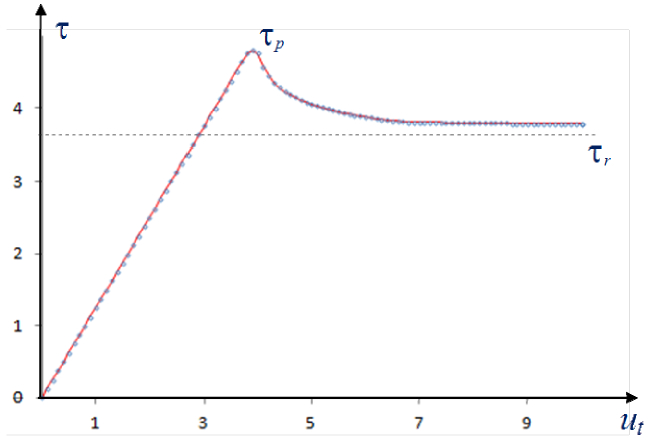
This evolution is controlled by the ductility parameter β. The evolution of g can include a hardening (increasing) stage if $\beta > 1$.

Non associate Plasticity with dilatancy angle ψ_0 : $$\underline{\dot{u}}^p = \dot{\lambda}\frac{\partial G}{\partial \underline{\sigma}}\ ,$$

$$G(\underline{\sigma},\xi) = \sqrt{\tau^2 + b^2 g^2} + h\,\sigma_n \tan\psi_0$$

Hardening law: $$\dot{\xi} = \alpha\left|\underline{\dot{u}}^p\right|$$





Nb = 12
Param1 = K_t (tangent stiffness)
Param2 = k_0 (normal stiffness)
Param3 = $K_{nt} = K_{tn}$ (non diagonal stiffness term causing dilatancy)
Param4 = e (maximum closure or physical thickness of the interface)
Param5 = C_0
Param6 = ϕ_0 (°)
Param7 = ψ_0 (°)
Param8 = σ_R
Param9 = g_r
Param10 = h_r
Param11 = α
Param12 = β

Condition: $\sigma_R \tan\phi_0 \leq C_0$
Internal Variables:
Vin(n,1): reserved for damage (not existing for this material)
Vin(n,2) : internal for non linear elasticity
Vin(n,3) =ξ

Non-linear elastoplastic joint with softening
lasticity

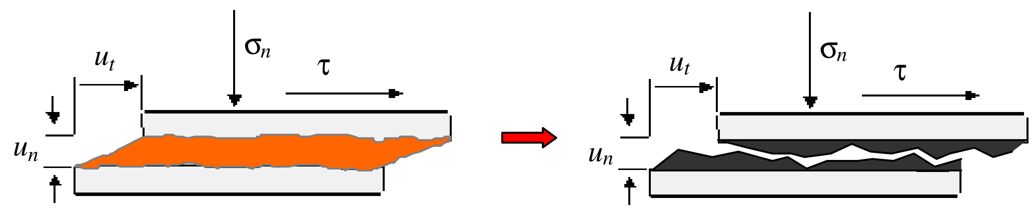
Param11 = α
Param12 = β

K_{tn}
aximum closure)

)
)

21510 : CZFrac: Cohesive Zone Fracture with Damage-Plasticity and Unilateral Contact

The Cohesive Zone Fracture (CZFrac) model describes the evolution of an interface from a cohesive interface like a rockjoint or thin layer of cohesive material (left), to a fracture with unilateral fictional contact (right).
The normal and tangent stiffnesses depend on a damage variable $0 \leq D \leq 1$ with residual values for D=1. The tangent relative displacement is divided into an elastic and a plastic part. The plastic part represents the irreversible slip on the frictional contact surface after the interface in totally damaged.

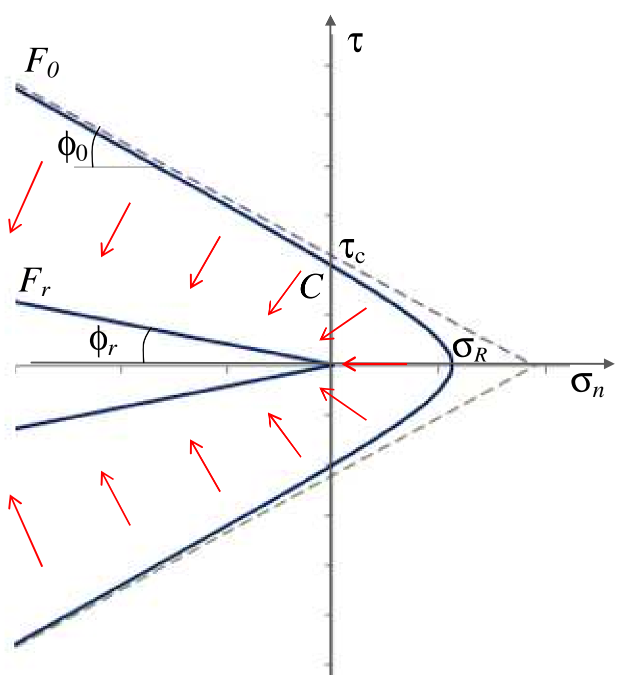


$D = 0$: cohesive joint $\qquad$ $D = 1$: frictional contact

$$\underline{\sigma} = \left[(1-D)\boldsymbol{K} + s\boldsymbol{k}_0\right]\left(\underline{u} - \underline{u}^p\right)$$

$$\boldsymbol{K} = \begin{bmatrix} K_t & 0 \\ 0 & K_n \end{bmatrix}$$

$$\boldsymbol{k}_0 = \begin{bmatrix} k_{0t} & 0 \\ 0 & \dfrac{k_{0n}}{1 + u_n / e} \end{bmatrix}$$



s : contact parameter depending on $-u_n$; $\qquad s = 1$ *if* $u_n < 0$, $\quad s = 0$ if $u_n \geq 0$

Damage criterion: $\quad F(\underline{\sigma}, D) = \tau^2 - \left(h\sigma_n \tan\phi\right)^2 + 2hg\tau_c\sigma_n\tan\phi - g^2C^2$

with: $\quad \tau_c = \dfrac{C^2 + \sigma_R^2 \tan^2\phi}{2\sigma_R \tan\phi}$, $\qquad h_r = \dfrac{\tan\phi_r}{\tan\phi}$

$$g(D) = (1-D)\left(1 - \beta\, ln(1-D)\right) \qquad h(D) = h_r + (1-D)^{\beta'}\left(1 - h_r\right)$$

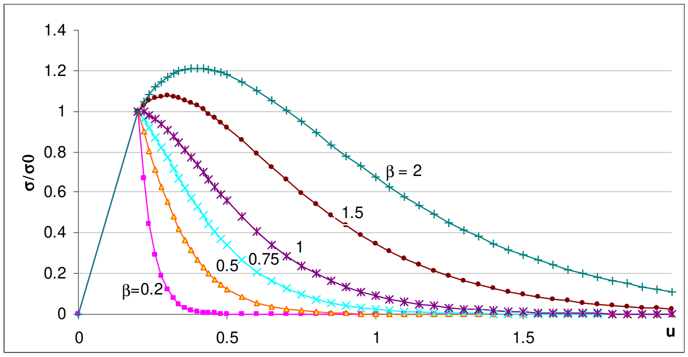
Plasticity: There is no plastic deformation as long as D is smaller than 1: $\dot{\underline{u}}^p = 0 \ \ if \ \ D < 1$

Then the plastic deformation occurs only in the shear direction: $\underline{u}^p = \begin{pmatrix} u_t^p \\ 0 \end{pmatrix}$

The plastic F^p criterion is the *residual* damage criterion, *i.e.*, the damage criterion for D=0. It is written as:

$$F^p(\underline{\sigma}) = |\tau| + h_r \sigma_n \tan\phi \qquad \text{with:} \quad \begin{pmatrix} \tau \\ \sigma_n \end{pmatrix} = s \begin{bmatrix} k_{0t} & 0 \\ 0 & \dfrac{k_{0n}}{1 + u_n / e} \end{bmatrix} \begin{pmatrix} u_t - u_t^p \\ u_n \end{pmatrix}$$

The parameter β > 0 controls the brittle (small β) to ductile (increasing β) damage behavior. For a pure normal stress, the normalized traction-separation curve has the following shape depending on β value:



Traction-Separation curves for different β values

Option Toughness

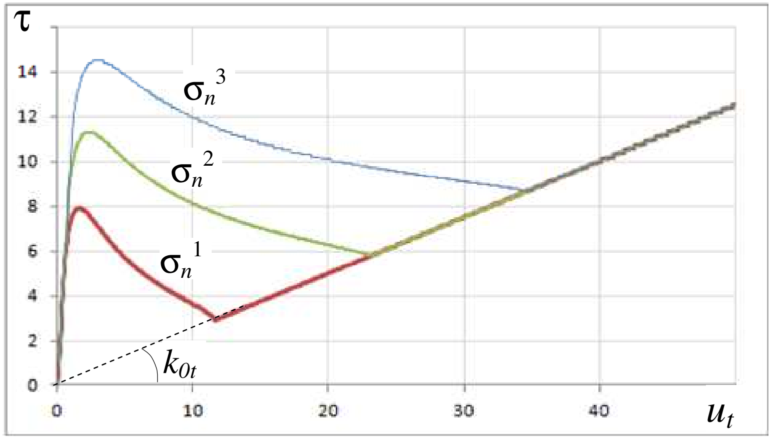
If the Toughness Option is chosen (Param13=1) then the parameters σ_R and C are determined from the toughness K^I_c (Param14) by the following relations depending on the size L of the joint element:

$$\sigma_R(L) = K_c^I \sqrt{\frac{2}{\pi L}}, \qquad C(L) = \frac{C}{\sigma_R} \sigma_R(L)$$

Where σ_R and C are the parameters 4 and 5 defined for the material (see the list below). This allows modeling well the propagation for large values of L without mesh size dependency.

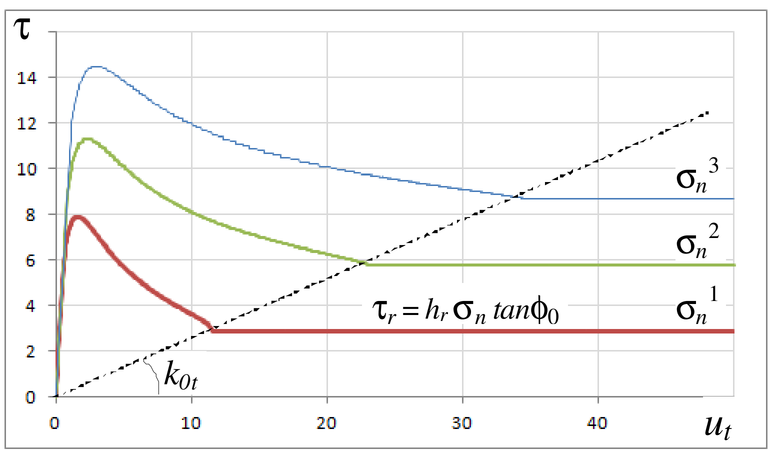
Option Plasticity

For a shear loading, the shear stress versus slip displacement has de same type of dependency on β value in damage phase. It depends also on the value of the normal stress and if plasticity is taken into account or not. If the plasticity is not modeled (Option 0 for the parameter 12 of the model), then the curve follows the line with the slope k_{0t} (residual tangent stiffness) for great values of u_t (following figure):



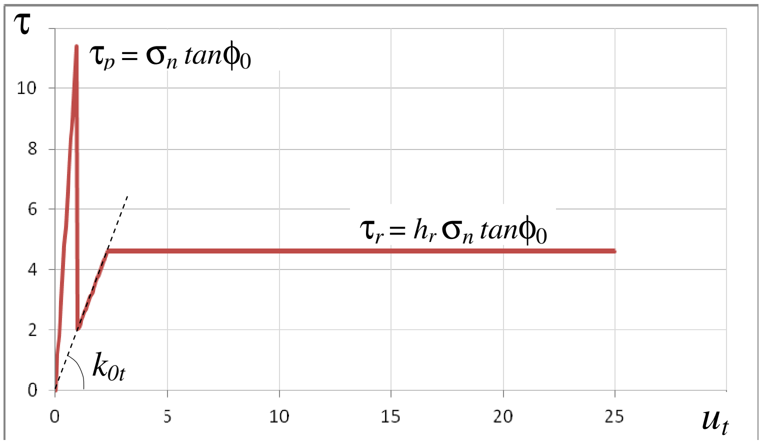
Shear stress versus slip under different compressive normal stresses for the option without plasticity: the curves join and follow the elastic line with residual stiffness k_{0t}

If plasticity is taken into account (Option 1 for the parameter 12 of the model), then the curve ends on a plastic plateau with the residual shear stress τ^r. This shear stress is related to the normal stress σ_n with the residual friction angle $h_r\,tan\phi_0$ (following figure).
Note that this relation holds only of the contact is maintained: because of the unilateral contact condition, if u_n>0 then s = 0 and then σ_n = 0 and so τ_r = 0.



Shear stress versus slip under different compressive normal stresses for the option with plasticity

For brittle damage (small values of β) and plasticity, it is possible to obtain a sharp decrease of the shear stress after the peak value and then an increase to reach the plastic residual stress (following figure).



Shear stress versus slip for the option with plasticity and brittle damage ($\beta < 1$)

Nb = 12
Param1 = K_t
Param2 = K_n
Param3 = e
Param4 = σ_R
Param5 = C
Param6 = ϕ (°)
Param7 = h_r
Param8 = β
Param9 = β'
Param10 = k_{0t}
Param11 = k_{0n}
Param12 = *Option* (1 if plasticity taken into account)
Internal variable
Vin(n,1) : D
Necessary Condition on parameters: $C > \sigma_R \; tan\; \phi$

21510	Cohesive Fracture with Damage-Plasticity and Unilateral Contact
Nb: 14	
Param1 = K_t	Param11 = k_{0n}
Param2 = K_n	Param12 = *Option Plasticity*
Param3 = e	Param13 = *Option Toughness*
Param4 = σ_R	Param14 = K^I_c
Param5 = C	
Param6 = ϕ (°)	
Param7 = h_r	
Param8 = β	
Param9 = β'	
Param10 = k_{0t}	

I.3) Mechanics - **BULK MATERIALS**

31100 : Linear elastic and isotropic material

Nb = 2
Param1 = E (Young's modulus)
Param2 = ν (Poissson's ratio)

31100 Linear Elastic and Isotropic Material
Nb : 2 Param1 = E (Young's modulus) Param2 = ν (Poisson's ratio)

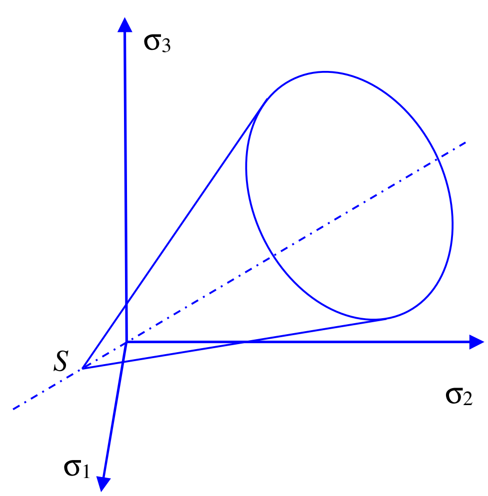
31110 : Elastic-plastic isotropy with Drucker-Pragar criterion

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

Linear elasticity with parameters E and ν (see the model 31100)
Plasticity with Drucker-Prager criterion : $F(\boldsymbol{\sigma}) = \sqrt{3J_2} + \sin\alpha\ I_1 - K$

$$I_1 = \sigma_{ii}\ , \quad S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}\ , \quad J_2 = \frac{1}{2}S_{ij}S_{ij}$$

K and $\sin\alpha$ are material constants.



Note that the Drucker-Prager criterion is basically written as:

$$F(\boldsymbol{\sigma}) = \sqrt{J_2} + \gamma\ I_1 - K'$$

The equivalence between the two expressions is ensured by taking:

$$\sin\alpha = \sqrt{3}\,\gamma\ , \quad K = \sqrt{3}K'$$

Nb = 4
Param1 = *E*
Param2 = ν
Param3 = *K*
Param4 = sinα

31110	Linear Isotropic Elasticity with Drucker-Prager Plastic Criterion

Nb : 4
Param1 = *E*
Param2 = ν
Param3 = *K*
Param4 = sinα

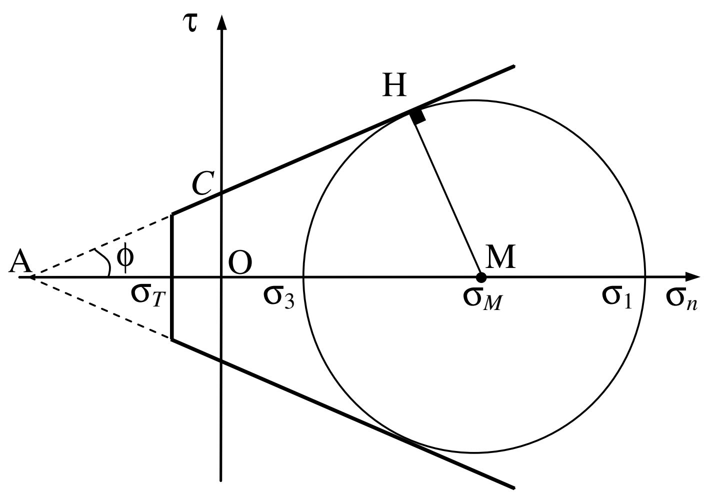
31120 : Elastic-plastic isotropic material with Mohr-Coulomb criterion, Non-Associate and Traction Truncated

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

Linear elasticity with parameters E and ν (see the model 31100)
Plasticity with Mohr-Coulomb criterion:

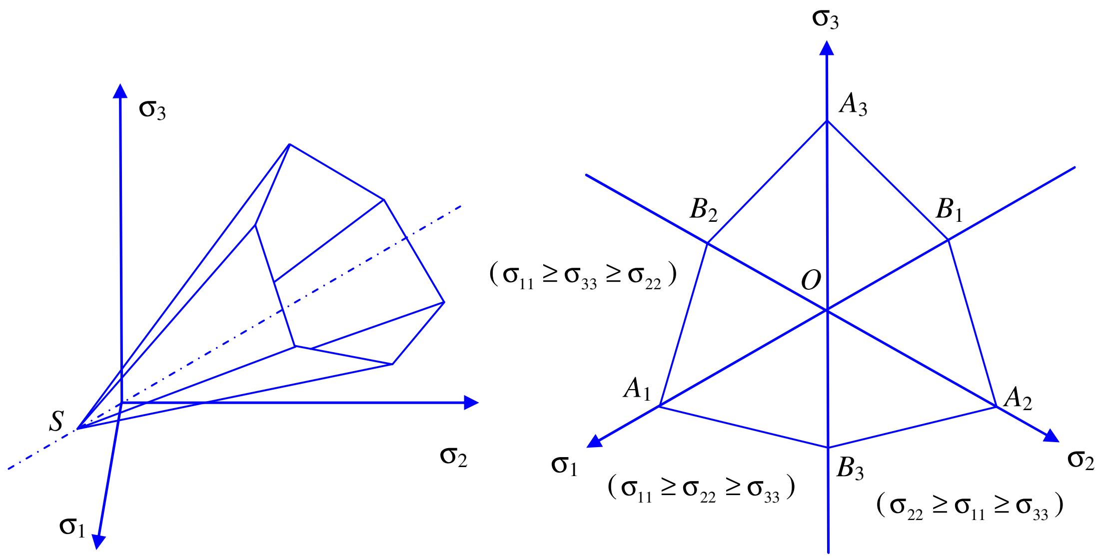
$F(\boldsymbol{\sigma}) = \dfrac{\sigma_1 - \sigma_3}{2} + \dfrac{\sigma_1 + \sigma_3}{2}\sin\phi - C\cos\phi \le 0$, where $\sigma_1 \ge \sigma_2 \ge \sigma_3$ principal stresses.

(In Disroc, compressions are negative, and the above model is equivalent to Soil Mechanics convention, where compressions are positive, and then the Mohr-Coulomb criterion reads $F(\boldsymbol{\sigma}) = \dfrac{\sigma_1 - \sigma_3}{2} - \dfrac{\sigma_1 + \sigma_3}{2}\sin\phi - C\cos\phi \le 0$, where $\sigma_1 \ge \sigma_2 \ge \sigma_3$ principal stresses, as in the following figure).



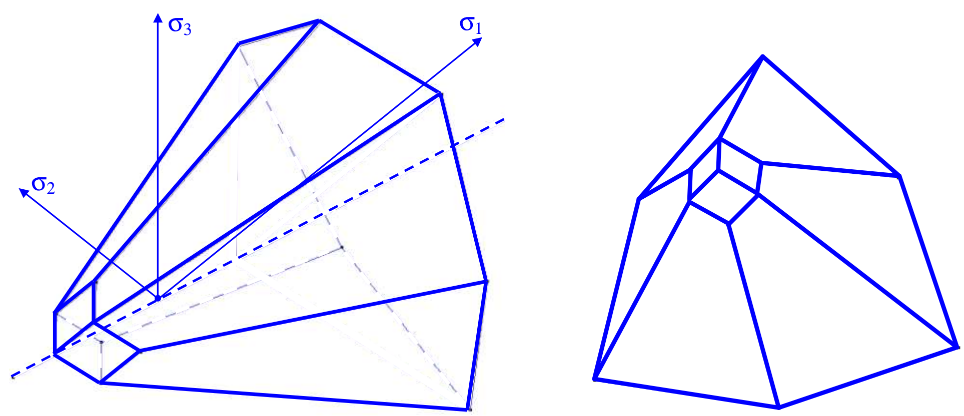
Flow rule: $\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\dfrac{\partial G}{\partial \boldsymbol{\sigma}}$ or more precisely $\dot{\boldsymbol{\varepsilon}}^p \in \dfrac{\partial G}{\partial \boldsymbol{\sigma}}$ (external normals cone for singular points, with:

Plastic Potential : $G(\boldsymbol{\sigma}) = \dfrac{\sigma_1 - \sigma_3}{2} + \dfrac{\sigma_1 + \sigma_3}{2}\sin\psi$



Mohr-Coulomb Criterion in principal stresses space

Tensile strength truncation: $\sigma_1 \leq \sigma_T$ where σ_T designates the tensile strength



Traction Truncated Mohr-Coulomb Criterion

Note 210514:

if $\sigma_T \geq C \dfrac{\cos\phi}{\sin\phi}$ then it has no effect. If $\dfrac{2C\cos\phi}{1+\sin\phi} \leq \sigma_T \leq C \dfrac{\cos\phi}{\sin\phi}$ then it has no effect for *uniaxial tractions*: the tensile unixial strength remains equal to $\dfrac{2C\cos\phi}{1+\sin\phi}$. If $\sigma_T \leq \dfrac{2C\cos\phi}{1+\sin\phi}$ then it will represent the limit aof uniaxial traction allowed by the criterion, or the tensile (uniaxial) strength.

Nb = 6
Param1 = E
Param2 = ν
Param3 = C
Param4 = ϕ (°)
Param5 = ψ (°)
Param6 = σ_T

31120 Linear Isotropic Elasticity with
Mohr-Coulomb Plastic Criterion
Non-Associate and Traction Truncated

Nb : 6
Param1 = E
Param2 = ν
Param3 = C
Param4 = ϕ (°)
Param5 = ψ (°)
Param6 = σ_T

31121 : Elastic-plastic Mohr-Coulomb with evolving properties (Gelisol)

This model is exactly the same that the Elastic-plastic Mohr-Coulomb model (31120) but with two set of parameters. The model 31120 includes 6 parameters (E,ν,C, ϕ, ψ, σ_T). The model 31120 includes 6 parameters (E_i,ν_i, C_i, ϕ_i, ψ_i, σ_i^T) for the initial values of the Young modulus, Poisson ratio, cohesion, friction angle, dilation angle and tensile strength, and 6 parameters (E_f,ν_f, C_f, ϕ_f, ψ_f, σ_f^T) for the initial values of these quantities. The evolution of the parameters between the initial and final values is given by internal variables v^i (i=1,6) in the following form :

$$E = (1-v_1)\,E_i + v_1\,E_f$$
$$\nu = (1-v_2)\,\nu_i + v_2\,\nu_f$$
$$C = (1-v_3)\,C_i + v_3\,C_f$$
$$\phi = (1-v_4)\,\phi_i + v_4\,\phi_f$$
$$\psi = (1-v_5)\,\psi_i + v_5\,\psi_f$$
$$\sigma_T = (1-v_6)\,\sigma^T{}_i + v_6\,\sigma^T{}_f$$

The evolution of the internal variable can be handled by the user in the User module. The default value of the internal variables is zero.
For Gelisol model conceived to model soil and rock freezing phenomenon and its effect on the mechanical properties, the evolution of internal variables is handled automatically in Disroc modules according to the constitutive equations of the coupled THM phenomena (See documentation on the Gelisol model).

Note 210515:
The Note 210514 for the material 31120 concerning the relation between the tensile strength truncation remains valid for the evolving parameters, *i.e.*, it takes into accound the evolving quantities and not the initial or finale values of the parameters. According to the evolution of the internal variables, the tensile strength σ_T can become greater or smaller than the limit given by the Mohr-Coulomb criterion and make that the truncation become active or not.

Note 210516:
The variation of the elastic parameters E and ν makes necessary the computation of the whole rigidity matrix at each time increment and this is a very time consuming action. If the difference between the initial and final values of these parameters is small, it is better to take the same values for them in order to reduce computation time. Disroc does not take into account the variation of the material's stiffness if it is less than 0.1 %, or more precisely if:

$$\frac{\left|E_f - E_i\right|}{\left(E_f + E_i\right)/2} + \left|\nu_f - \nu_i\right| < 0.001$$

In this case Disroc considers E and ν constants and equal to E_i and ν_i, and the compotation becomes faster.

Internal Variables:
Vin(n,1): v_1 (can represent also scalar damage)
Vin(n,2) : v_2

Vin(n,3): v_3
Vin(n,4) : v_4
Vin(n,5) : v_5
Vin(n,6) : v_6

Nb = 12

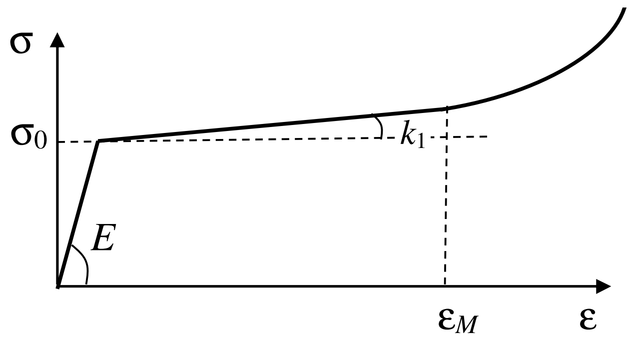
Param1 = E_i	*Initial Young modulus*
Param2 = ν_I	*Initial Poisson ration*
Param3 = C_i	*Initial Cohesion*
Param4 = ϕ_i (°)	*Initial Friction Angle*
Param5 = ψ_i (°)	*Initial Dilation Angle*
Param6 = $\sigma^T{}_i$	*Initial Tensile Strength*
Param7 = E_f	*Final Young modulus*
Param8 = ν_f	*Final Poisson ration*
Param9 = C_f	*Final Cohesion*
Param10 = ϕ_f (°)	*Final Friction angle*
Param11= ψ_f (°)	*Final Dilation angle*
Param12 = $\sigma^T{}_f$	*Initial Tensile Strength*

31121 Evolving Elastoplastic Mohr-Coulomb Gelisol

Nb : 12
Param1 = E_i
Param2 = ν_i
Param3 = C_i
Param4 = ϕ_i (°)
Param5 = ψ_i (°)
Param6 = $\sigma^T{}_i$
Param7 = E_f
Param8 = ν_f
Param9 = C_f
Param10 = ϕ_f (°)
Param11= ψ_f (°)
Param12 = $\sigma^T{}_f$

31125 : Elastic-plastic Mohr-Coulomb with Compressibility Cap (MC-CAP)

The objective of this model is to produce a compressible material with the following shape of the stress-strain curve for uniaxial compression as well as oedometric compression:



The curve presents, after the elastic limit, a long plastic stage with zero to small slope followed by a quickly increasing slope.

The Mohr-Coulomb criterion with traction cutoff is used as well as a compressible material criterion, both with hardening (following figure).

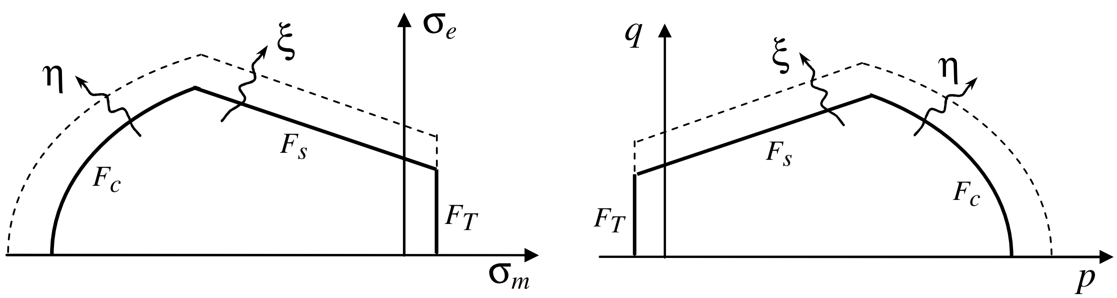


Figure: F_s : Shear criterion (Mohr-Coulomb), F_T : Traction cutoff, F_c : Compressibility
($q = \sigma_e, \quad p = -\sigma_m$)

Constitutive model:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

Elasticity: $$\dot{\boldsymbol{\varepsilon}}^e = \frac{1+\nu}{E}\dot{\boldsymbol{\sigma}} - \frac{\nu}{E} tr(\dot{\boldsymbol{\sigma}})\,\boldsymbol{\delta}\,,$$

Plasticity: $$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}_s \frac{\partial G_s}{\partial \boldsymbol{\sigma}} + \dot{\lambda}_T \frac{\partial G_T}{\partial \boldsymbol{\sigma}} + \dot{\lambda}_c \frac{\partial G_c}{\partial \boldsymbol{\sigma}}$$

Flow rule: $$\begin{array}{l} if \;\; F_i < 0 \;\; then \;\; \dot{\lambda}_i = 0 \\ if \;\; F_i = 0 \;\; then \;\; \dot{\lambda}_i \geq 0\,,\; \dot{F}_i \leq 0\,,\; \dot{\lambda}_i \dot{F}_i = 0 \end{array}$$ for $i=s$, c or T

I) Mohr-Coulomb + Traction Cutoff

This part of the model corresponds to the model 31120 with hardening for the parameter cohesion. The traction cutoff value remains constant (no hardening).
With negative compression sign convention, the Mohr-Coulomb criterion reads:

$$F_s(\boldsymbol{\sigma},\xi) = (\sigma_1 - \sigma_2) + (\sigma_1 + \sigma_2)\sin\phi - 2C(\xi)\cos\phi \leq 0$$

Or:

$$F_s(\boldsymbol{\sigma},\xi) = (\sigma_1 - \sigma_2) + (\sigma_1 + \sigma_2)\sin\phi - R_c(\xi)\,(1-\sin\phi) \leq 0$$

$$R_c = \frac{2C\cos\phi}{1-\sin\phi}$$

Where C is the cohesion, R_c the uniaxial compression strength (UCS) and ξ a hardening parameter.

$$R_c(\xi) = R_c^0 + k_1\xi + k_2\left\langle \xi - \varepsilon_0^p \right\rangle^2$$

where the symbol $\langle\, . \,\rangle$ stands for the positive part:

$$< x > = 0 \quad if \quad x < 0$$
$$< x > = x \quad if \quad x \geq 0$$

Plastic potential: $G_s(\boldsymbol{\sigma},\xi) = (\sigma_1 - \sigma_2) + (\sigma_1 + \sigma_2)\sin\psi$

The hardening rule is:

$$\dot{\xi} = \alpha\sqrt{\dot{\boldsymbol{\varepsilon}}^p : \dot{\boldsymbol{\varepsilon}}^p} \;, \quad \alpha = \frac{1-\sin\psi}{\sqrt{2\left(1+\sin^2\psi\right)}}$$

The value of α assures that for a uniaxial compression we have : $\delta\xi = \left|\delta\varepsilon_{yy}^p\right|$.

II) Compressible material

This mechanism of plastic deformation is defined by the following equations:

Plastic criterion $F_c(\boldsymbol{\sigma},\eta) = \sqrt{a_2\sigma_e^2 + \varphi\sigma_m^2} - (1 - a_1\varphi)\,\sigma_M(\eta)$

Plastic potential $G_c(\boldsymbol{\sigma},\eta) = \sqrt{a_3\sigma_e^2 + \varphi\sigma_m^2}\;, \quad \dot{\boldsymbol{\varepsilon}}_c^p = \dot{\lambda}_c \dfrac{\partial G_c}{\partial \boldsymbol{\sigma}}$

Hardening $\sigma_M(\eta) = \sigma_M^0 + k_3\eta + k_4\left\langle \eta - \varepsilon_v^p \right\rangle^2$

Hardening law $\dot{\eta} = -\dot{\boldsymbol{\varepsilon}}^p : \boldsymbol{\delta}$

Where σ_e is the Mises Equivalent Stress and σ_m the mean stress:

$$\sigma^e = \sqrt{\frac{3}{2}S_{ij}S_{ij}} \;, \qquad \sigma_m = \frac{1}{3}\boldsymbol{\sigma} : \boldsymbol{\delta}$$

φ is the porosity and a_1, a_2, σ_M^0, k_3, k_4 and ε_0^v are constant parameters. The evolution of the porosity versus des total volumetric strain is given by:

$$\dot{\varphi} = (1-\varphi)\dot{\varepsilon}_p^{vol}$$

Which supposes that the solid grains incompressible and that the elastic volumetric strain is negligible. This equation can be integrated in:

$$\varphi = 1 - (1-\varphi_0)e^{-\varepsilon_p^{vol}}$$

With φ_0 the initial value of the porosity for ε=0. Of course, the condition $\varphi \geq 0$ should be verified. To avoid numerical problems with φ=0¸the condition of $\varphi \geq 0.0001$ will be imposed when using this equation.

Note : When a uniaxial compression σ is considered, the condition that MC criterion be reached is that $\sigma = R_c$ and the condition that the elliptic cap criterion be reached is that:

$$\sigma = \frac{1 - a_1\varphi}{\sqrt{a_2 + \varphi / 9}}\sigma_M$$

So the condition that for a uniaxial compression the MC criterion be reached before the elliptical cap is:

$$\frac{1 - a_1\varphi_0}{\sqrt{a_2 + \varphi_0 / 9}}\sigma_M^0 > \frac{2C\cos\phi}{1 - \sin\phi}$$

Parameters

Nb = 17

Param1 = E *Young modulus*

Param2 = ν *Poisson ration*

Param3 = C *Initial Cohesion*

Param4 = ϕ (°) *Friction Angle*

Param5 = ψ (°) *Dilation Angle*

Param6 = σ^T *Tensile Strength*

Param7 =$\sigma^0{}_M$ *Limit compression stress* (positive for compression)

Param8 = φ_0 *Initial porosity* ($0.0001 < \varphi_0 < 1$)

Param9 = a_1 *positive number*

Param10 = a_2 *positive number*

Param11 = a_3 *positive number*

Param12 = k_1

Param13 = k_2

Param14 = $\varepsilon^p{}_a$: *Limit axial plastic strain for linear hardening* (positive for compression)

Param15 = k_3

Param16 = k_4

Param17 = $\varepsilon^p{}_v$: *Limit plastic strain for linear hardening* (positive for compression)

Internal variable

$V_{in,m}(n,1)$: φ (porosity)

$V_{in,m}(n,2)$: ξ

$V_{in,m}(n,3)$: η

Conditions to be satisfied: $0 < 1\text{-}a_1\varphi_0$

31125 Elastoplastic Mohr-Coulomb with compressibility cap (MC-CAP)

Nb : 17

Param1 = E	Param13 = k_2
Param2 = ν	Param14 = ε^p_a
Param3 = C	Param15 = k_3
Param4 = ϕ (°)	Param16 = k_4
Param5 = ψ (°)	Param17 = ε^p_v
Param6 = σ^T	
Param7 = σ^0_M	
Param8 = φ_0	
Param9 = a_1	
Param10 = a_2	
Param11 = a_3	
Param12 = k_1	

31130 : Viscoelastic isotropic material : Linear elasticity and Norton-Hoff creep law

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^v$$

$$\dot{\boldsymbol{\varepsilon}}^e = \frac{1+\nu}{E}\dot{\boldsymbol{\sigma}} - \frac{\nu}{E} tr(\dot{\boldsymbol{\sigma}})\,\boldsymbol{\delta}\,, \qquad \dot{\boldsymbol{\varepsilon}}^v = \frac{3}{2}\alpha\,\xi^{\alpha-1}\dot{\xi}\,\frac{\boldsymbol{S}}{\sigma_e}$$

with:

$\boldsymbol{S}$ stress deviator, $S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$, Mises equivalent stress $\sigma_e = \sqrt{3J_2}$, $J_2 = \frac{1}{2}S_{ij}S_{ij}$

$$\dot{\xi} = \left(a < \sigma_e - \sigma_c >^n\right)^{1/\alpha}$$

where, the *positive part* function <·> is defined as:

$$< x > = 0 \quad if \quad x < 0$$
$$< x > = x \quad if \quad x \geq 0$$

To avoid numerical problems near $\xi = 0$, the law is completed by:

$$\dot{\boldsymbol{\varepsilon}}^v = \frac{3}{2}\alpha\,\varepsilon_0^{\alpha-1}\dot{\xi}\,\frac{\boldsymbol{S}}{\sigma_e} \quad \text{if} \quad \xi^\alpha \leq \varepsilon_0$$

If $\alpha = 1$, the Norton-Hoff creep model is recovered. For a uniaxial stress the Lemaitre creep law is found: $\varepsilon(t) = a\left\langle \sigma - \sigma_c \right\rangle^n t^\alpha$

The four parameters a, n, α, σ_c can thus be identified from uniaxial creep results.

Nb = 7
Param1 = E
Param2 = ν
Param3 = a (attention to the stress and time unities)
Param4 = n
Param5 = α
Param6 = σ_c
Param7 = ε_0

Internal Variables:
Vin(n,1): reserved for damage (not existing for this material)
Vin(n,2) : ξ, internal

31130	Linear Isotropic Elasticity with Lemaitre-Norton-Hoff Creep law
Nb : 7	
Param1 = E	
Param2 = ν	
Param3 = a	(attention to the stress and time unities)
Param4 = n	
Param5 = α	
Param6 = σ_c	
Param7 = ε_0	

31140 : Viscoplastic isotropic material : Linear Elasticity & Associate Plasticity with Mises criterion and Kinematic + isotropic hardening & Lemaitre viscoelasticity

Total strain: $\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p + \dot{\boldsymbol{\varepsilon}}^v$

Elasticity: $\dot{\boldsymbol{\varepsilon}}^e = \dfrac{1+\nu}{E}\dot{\boldsymbol{\sigma}} - \dfrac{\nu}{E} tr(\dot{\boldsymbol{\sigma}})\,\boldsymbol{\delta}$,

Plasticity: $\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\dfrac{\partial F}{\partial \boldsymbol{\sigma}}$, $\begin{array}{l} if\ F<0\ then\ \ \dot{\lambda}=0 \\ if\ F=0\ then\ \ \dot{\lambda}\geq 0,\ \dot{F}\leq 0,\ \dot{\lambda}\dot{F}=0 \end{array}$

with: $F(\boldsymbol{\sigma}, \boldsymbol{X}, R) = \sqrt{J_2(\boldsymbol{\sigma}-\boldsymbol{X})} - K$

$$J_2(\boldsymbol{\sigma}-\boldsymbol{X}) = \frac{1}{2}S'_{ij}S'_{ij}, \quad S'_{ij} = \sigma_{ij} - X_{ij} - \frac{1}{3}(\sigma_{kk}-X_{kk})\delta_{ij} \qquad (X_{kk}=0)$$

$$K = K_0 + K_1\left(1-e^{-bp}\right) \qquad \dot{p} = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^p : \dot{\boldsymbol{\varepsilon}}^p}$$

$$\boldsymbol{X} = \boldsymbol{X_1} + \boldsymbol{X_2}, \qquad \dot{\boldsymbol{X}}_1 = c_1\,\dot{\boldsymbol{\varepsilon}}^p - d_1\dot{p}\boldsymbol{X}_1, \qquad \dot{\boldsymbol{X}}_2 = c_2\,\dot{\boldsymbol{\varepsilon}}^p - d_2\dot{p}\boldsymbol{X}_2,$$

For the initial state of the material, p=0 and $\boldsymbol{X_1} = \boldsymbol{X_2} = 0$.

Viscous deformation: $\dot{\boldsymbol{\varepsilon}}^v = \dfrac{3}{2}\alpha\,\xi^{\alpha-1}\dot{\xi}\,\dfrac{\boldsymbol{S}}{\sigma_e}$

with: Mises equivalent stress $\sigma_e = \sqrt{\dfrac{3}{2}S_{ij}S_{ij}}$, $\qquad S_{ij} = \sigma_{ij} - \dfrac{1}{3}\sigma_{kk}\delta_{ij}$

$$\xi = \left(a < \sigma_e - \sigma_c >^n\right)^{1/\alpha}$$

Where σ_c is a stress threshold and <·> represents the *positive part* function:

$$\begin{array}{lll} <x> = 0 & if & x<0 \\ <x> = x & if & x\geq 0 \end{array}$$

To avoid numerical problems near $\xi = 0$, the law is completed by: $\dot{\boldsymbol{\varepsilon}}^v = \dfrac{3}{2}\alpha\,\varepsilon_0^{\alpha-1}\dot{\xi}\,\dfrac{\boldsymbol{S}}{\sigma_e}$ if $\xi^\alpha \leq \varepsilon_0$.

For a uniaxial stress the creep law becomes (Lemaitre creep law with stress threshold):

$$\varepsilon(t) = a\left\langle \sigma - \sigma_c \right\rangle^n t^\alpha$$

The four parameters a, n, α, σ_c can thus be identified from uniaxial creep results.

If α =1, the Norton-Hoff creep model is recovered: $\dot{\boldsymbol{\varepsilon}}^v = \dfrac{3}{2}a\left\langle \sigma_e - \sigma_c \right\rangle^n \dfrac{\boldsymbol{S}}{\sigma_e}$

Number of parameters 14:

Nb = 14
Param1 = E
Param2 = ν
Param3 = K_0
Param4 = K_1
Param5 = b
Param6 = c_1
Param7 = d_1
Param8 = c_2
Param9 = d_2
Param10 = a (attention to the stress and time unities)
Param11 = n
Param12 = α
Param13 = σ_c
Param14 = ε_0

Internal Variables: 9
Vin(n,1): reserved for damage (not existing for this material)
Vin(n,2) : Vin(n,3), Vin(n,4): X^1_{xx}, X^1_{yy}, X^1_{xy} ($X^1_{zz}= -X^1_{xx}- X^1_{yy}$)
Vin(n,5) : Vin(n,6), Vin(n,7): X^2_{xx}, X^2_{yy}, X^2_{xy} ($X^2_{zz}= -X^2_{xx}- X^2_{yy}$)
Vin(n,8) : p
Vin(n,9) : ξ, internal

31140	Lemaitre-Chaboche Elastic-Plastic with Lemaitre-Norton-Hoff Creep
Nb : 14	
Param1 = E	Param11 = n
Param2 = ν	Param12 = α
Param3 = K_0	Param13 = σ_c
Param4 = K_1	Param14 = ε_0
Param5 = b	
Param6 = c_1	
Param7 = d_1	
Param8 = c_2	
Param9 = d_2	
Param10 = a	

31200 : Linear Elasticity with General Anisotropy

In 2D plane problems, $\varepsilon_{13}=\varepsilon_{23}=\sigma_{13}=\sigma_{23}=0$, and the Hook law reduces to :

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12} & c_{13} & c_{16}\\ & c_{22} & c_{23} & c_{26}\\ & & c_{33} & c_{36}\\ & & & c_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{12}\end{bmatrix}$$

The elastic parameters, in the more general case of anisotropy are the 10 followings:

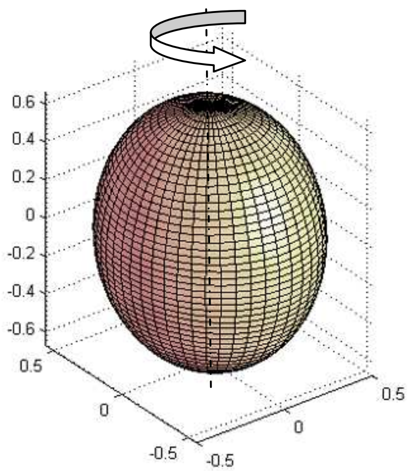
Nb = 10
Param1 = c_{11}, Param2 = c_{12}, Param3 = c_{13}, Param4 = c_{16},
Param5 = c_{22}, Param6 = c_{23}, Param7 = c_{26},
Param8 = c_{33}, Param9 = c_{36},
Param10= c_{66}

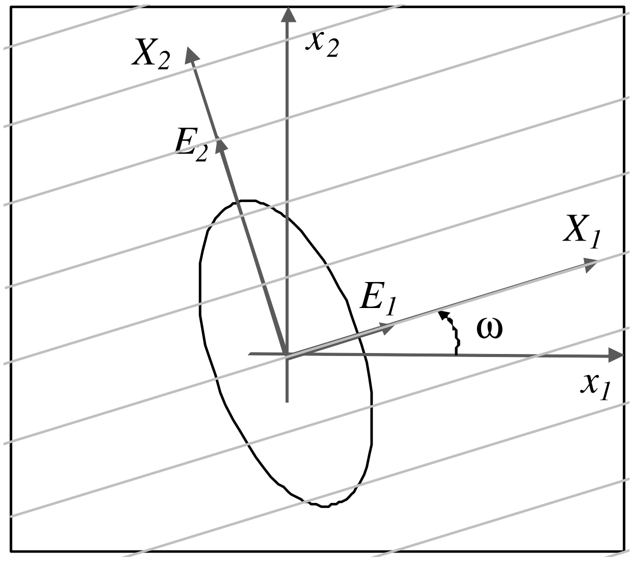
31200	Linear Elasticity with General Anisotropy
Nb : 10 Param1 = c_{11} Param2 = c_{12} Param3 = c_{13} Param4 = c_{16} Param5 = c_{22} Param6 = c_{23} Param7 = c_{26} Param8 = c_{33} Param9 = c_{36} Param10= c_{66}	

31300 : Linear elasticity with Saint Venant anisotropy

The Saint Venant ellipsoïdal material (Pouya 2007) is a 3D anisotropic material depends on four parameters, three Young's modulus (E_1, E_2, E_3) and Poisson ration ν.

The basic assumption is that the Young's modulus in different directions varies in special way making that indicator surface of its fourth root is a spheroid. The tensor ***s*** and ***c*** defined by:





$$
\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{36} \\ c_{16} & c_{26} & c_{36} & c_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \end{bmatrix} , \qquad \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{16} \\ s_{12} & s_{22} & s_{23} & s_{26} \\ s_{13} & s_{23} & s_{33} & s_{36} \\ s_{16} & s_{26} & s_{36} & s_{66} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \end{bmatrix}
$$

have the following expressions:

$$
\boldsymbol{s} = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu}{\sqrt{E_1E_2}} & \frac{-\nu}{\sqrt{E_1E_3}} & & & \\ \frac{-\nu}{\sqrt{E_1E_2}} & \frac{1}{E_2} & \frac{-\nu}{\sqrt{E_2E_3}} & & & \\ \frac{-\nu}{\sqrt{E_1E_3}} & \frac{-\nu}{\sqrt{E_2E_3}} & \frac{1}{E_3} & & & \\ & & & \frac{2(1+\nu)}{\sqrt{E_2E_3}} & & \\ & & & & \frac{2(1+\nu)}{\sqrt{E_3E_1}} & \\ & & & & & \frac{2(1+\nu)}{\sqrt{E_1E_2}} \end{bmatrix}
$$

$$
\boldsymbol{c} = \frac{1}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu)E_1 & \nu\sqrt{E_1E_2} & \nu\sqrt{E_1E_3} & & & \\ \nu\sqrt{E_1E_2} & (1-\nu)E_2 & \nu\sqrt{E_2E_3} & & & \\ \nu\sqrt{E_1E_3} & \nu\sqrt{E_2E_3} & (1-\nu)E_3 & & & \\ & & & \frac{1-2\nu}{2}\sqrt{E_2E_3} & & \\ & & & & \frac{1-2\nu}{2}\sqrt{E_1E_3} & \\ & & & & & \frac{1-2\nu}{2}\sqrt{E_1E_2} \end{bmatrix}
$$

If two elastic modulus are equal, for instance E_1=E_3, then a special case of transverse isotropy around the x_2–axis is found (Figure) depending on only three parameters (E_1, E_2, ν).

The model can include a rotation ω of X_2-axis, representing the direction with the Young's modulus E_2, with respect to the x_2-axis in the plane of calculation (x_1,x_2). Note that the out-of-plane modulus E_3 will be equal to E_1.

Nb = 5
Param1 = E_1
Param2 = E_2
Param3 = E_3
Param4 = ν
Param5 = ω (in degrees)

31300	Linear Elasticity with Saint Venant Ellipsoidal Anisotropy
Nb : 5	
Param1 = E_1 Param2 = E_2 Param3 = E_3 Param4 = ν Param5 = ω (in degrees)	

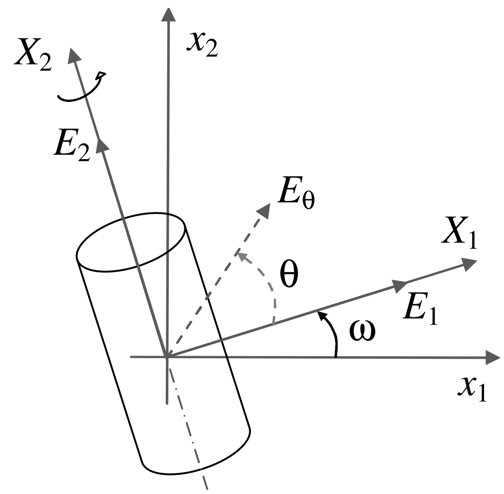
31400 : Linear elasticity with transverse isotropy

The elasticity of the material has axial symmetry around the X_2-axis. The axial Young's modulus is E_2 and the transverse one is E_1. The elastic tensor is defined by five independent parameters E_1, E_2, ν_{12}, ν_{13}, μ_{12} with the following complementary conditions:

$$E_3 = E_1\ ,\ \nu_{32} = \nu_{12}\ ,\ \nu_{31} = \nu_{13}\ ,$$

The constitutive equation in a coordinate system with x_2-axis superposed to the axis of symmetry X_2 reads :

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{12}}{E_1} & \frac{-\nu_{13}}{E_1} & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{12}}{E_1} & 0 \\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{12}}{E_1} & \frac{1}{E_1} & 0 \\ 0 & 0 & 0 & \frac{1}{\mu_{12}} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \end{bmatrix}$$



The model can include a rotation ω of X_2 with respect to the x_2-axis in the plane of calculation (x_1,x_2). Note that the out-of-plane modulus E_3 will be equal to E_1.

Note that the Young's modulus in a direction in the radial plane (X_1, X_2) and making an angle θ with X_1 (see the figure) is given by:

$$\frac{1}{E_\theta} = \frac{\cos^4\theta}{E_1} + (\frac{1}{\mu_{12}} - \frac{2\nu_{12}}{E_1})\cos^2\theta\sin^2\theta + \frac{\sin^4\theta}{E_2}$$

For identification of the parameters from test data, note that a coefficient ν_{21} different from ν_{12} could be defined for this material satisfying the symmetry condition:

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}$$

The coefficient ν_{21} can be measured in the following way: a uniaxial compression σ_{22} is applied in the direction X_2 and the strains ε_{22} and ε_{11} are measured respectively in axial and radial directions X_2 and X_1. Then $\nu_{21} = -\varepsilon_{11}/\varepsilon_{22}$ and ν_{21} is obtained from the above symmetry condition. It would be possible also to apply the uniaxial compression σ_{11} in direction X_1 and measure the strains ε_{11} and ε_{22} in directions X_1 and X_2. The problem in this case has not axial symmetry. But we get directly $\nu_{12} = -\varepsilon_{22}/\varepsilon_{11}$. No difference is to be considered for ν_{31} and ν_{13}.

Nb = 6
Param1 = E_1
Param2 = E_2
Param3 = ν_{12}

Param4 = ν_{13}
Param5 = μ_{12}
Param6 = ω (in degrees)

31400 Linear Elasticity with Transverse Isotropy

Nb: 6
Param1 = E_1
Param2 = E_2
Param3 = ν_{12}
Param4 = ν_{13}
Param5 = μ_{12}
Param6 = ω (in degrees)

31410 : Linear elasticity with transverse isotropy + Drucker-Prager plastic criterion

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

$$F(\boldsymbol{\sigma}) = \sqrt{J_2} + \gamma I_1 - K$$

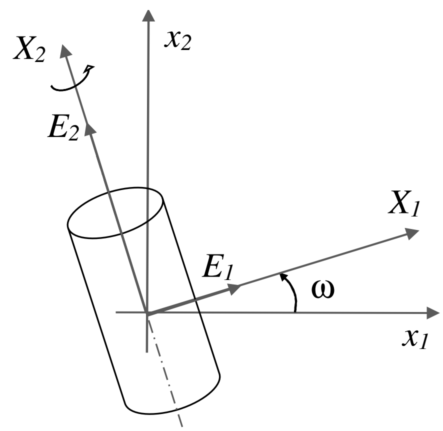
Elasticity : the same model than 31400 : transverse isotropy
Plasticity : the same model than 31110 : Drucker-Prager

Nb = 8
Param1 = E_1
Param2 = E_2
Param3 = ν_{12}
Param4 = ν_{13}
Param5 = μ_{12}
Param6 = ω (in degrees)
Param7 = K
Param8 = $\sin\alpha$

31410	Linear Elasticity with Transverse Isotropy and Drucker-Prager Plastic Criterion
Nb: 8 Param1 = E_1 Param2 = E_2 Param3 = ν_{12} Param4 = ν_{13} Param5 = μ_{12} Param6 = ω (in degrees) Param7 = K Param8 = $\sin\alpha$	

31430 : ANELVIP: Anisotropic elasto-viscoplastic material : Transverse Isotropic elasticity, anisotropic Mohr-Coulomb or Drucker-Prager plasticity and Lemaitre creep law

This elasto-visco-plastic material has axial symmetry around an axis related to the material and designated by X_2 (Figure). This material axis can make an angle ω with the x_2-axis of coordinate system. The elastic behavior corresponds to the general transverse isotropic material around the axis X_2 of the material with five independent parameters. The anisotropic plastic and viscous deformations of the material are defined by a linear transformation from isotropic plastic and viscous deformation models. They have also transverse isotropy around the axis X_2.



Constitutive model:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p + \dot{\boldsymbol{\varepsilon}}^v \qquad (1.1)$$

Elasticity: $$\dot{\boldsymbol{\varepsilon}}^e = \mathbb{C}^{-1} : \dot{\boldsymbol{\sigma}} \qquad (1.2)$$

Plasticity: $$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda} \frac{\partial \tilde{G}}{\partial \boldsymbol{\sigma}} \quad , \qquad \dot{\lambda} = 0 \ \ if \quad \tilde{F}(\boldsymbol{\sigma}) < 0 \qquad (1.3)$$

Creep: $$\dot{\boldsymbol{\varepsilon}}^v = \frac{3}{2} \alpha \, \xi^{\alpha-1} \dot{\xi} \, \frac{\tilde{S}^v}{\tilde{\sigma}_e^v} \ , \quad \dot{\xi} = \left(a \, \beta^v < \tilde{\sigma}_e^p - \sigma_c >^n \right)^{1/\alpha} \qquad (1.4)$$

With $\mathbb{C}$ the elastic tensor with transverse isotropy, $\tilde{F}$ anisotropic Mohr-Coulomb (traction truncated) or Drucker-Prager criterion and non-associated potential $\tilde{G}$ and anisotropic Norton-Lemaitre creep law with stress threshold obtained by transformation of isotropic material. The creep part or the plastic part of the model can be excluded to obtain a simple elastoplastic or a simple viscoelastic model.

Transformation

The material is supposed to be transverse isotropic with the axis of isotropy lying in the plane of modeling (x_1, x_2). This axis is represented by X_2 in the figure.

The direction dependency of the strain rate and of the stress threshold for plastic and viscous strain is defined by introducing a transformed $\tilde{\boldsymbol{\sigma}}$ obtained by as a linear function of $\boldsymbol{\sigma}$. This transformation is defined in the following way in the (X_1, X_2) coordinates:

$$\sigma = \begin{bmatrix} \sigma_{XX} & \sigma_{XY} & 0 \\ \sigma_{XY} & \sigma_{YY} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \rightarrow \tilde{\sigma} = \begin{bmatrix} \sigma_{XX} & f_T\sigma_{XY} & 0 \\ f_T\sigma_{XY} & f_N\sigma_{YY} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \qquad (1.5)$$

- A uniaxial stress in direction X_1, or any direction perpendicular to X_2 is not changed (X_2 remains an axis of symmetry)
- A uniaxial stress σ in direction X_2 is changed in a uniaxial stress $f_2\,\sigma$
- A pure shear stress τ in direction X_1X_2 is changed in a pure shear stress $f_T\,\tau$ in the same direction.

  We put:

$$f_N = 1 + a_N \quad , \quad f_T = \sqrt{f_N + b_T} \qquad (1.6)$$

The constants a_N and b_T are considered as two material's parameters describing its anisotropy. We note also:

$$f_T = 1 + a_T \qquad (1.7)$$

with the following relations:

$$a_T = \sqrt{1 + a_N + b_T} - 1 \ , \quad b_T = f_T^2 - f_N = a_T\left(2 + a_T\right) - a_N \qquad (1.8)$$

We note $\tilde{\sigma}^p$ the transformed stress $\tilde{\sigma}$ obtained with the parameters $(a_N^p\,,\ a_T^p)$ and $\tilde{\sigma}^v$ obtained with $(a_N^v\,,\ a_T^v)$.

We note $\tilde{S}$ and $\tilde{\sigma}_e$ the deviator stress and Mises equivalent stress associated to $\tilde{\sigma}$ and define:

$$\beta = \frac{\tilde{\sigma}_e}{\sigma_e} \qquad (1.9)$$

For a *uniaxial stress* in the direction θ with respect to the x_1, the ratio β has the following expression:

$$\beta(\overline{\theta}) = \sqrt{\left(1 + a_N \sin^2\overline{\theta}\right)^2 + 3b_T \sin^2\overline{\theta}\cos^2\overline{\theta}} \qquad (1.10)$$

Where:

$$\overline{\theta} = \theta - \omega \qquad (1.11)$$

The transformation applied to the viscous strain $\tilde{\varepsilon} = \beta^v\,\varepsilon$ allows making the creep law anisotropic (Figure). But note that a uniaxial stress σ in a direction θ different of ω is not transformed to a uniaxial stress and so different β rations are obtained for UCS or for the creep rate as it will be seen below.

I) Elasticity

The elastic behavior has axial symmetry or the transverse isotropy around the axis X_2 (see the figure). The Young's modulus in direction X_2 is E_2 and in directions X_1 and X_3 (out of plane), equal to E_1. The three other parameters are the Poisson's ratios ν_{12} and ν_{13} and the shear modulus μ_{12}. The elastic model here is exactly the same that the model 31400 with the five parameters (E_1, E_2, ν_{12}, ν_{13}, μ_{12}) and the angle ω between the axis of symmetry X_2 and the coordinate axis x_2. See the material 31400 for the method of identification of parameters and the Young's modulus in different directions of the material.

II) Plastic deformation

The plastic deformation is defined by the plastic criterion $\tilde{F}$ and the plastic potential $\tilde{G}$ with the following relations:

$$\tilde{F}(\boldsymbol{\sigma}) = F(\tilde{\boldsymbol{\sigma}}^p) \ , \qquad \tilde{G}(\boldsymbol{\sigma}) = G(\tilde{\boldsymbol{\sigma}}^p) \tag{1.12}$$

Where the transformed stress $\tilde{\boldsymbol{\sigma}}^p$ is deduced from $\boldsymbol{\sigma}$ with the set of parameters $(a_N^p\ ,\ a_T^p)$. The plastic yield rule reads:

$$\tilde{F}(\boldsymbol{\sigma}) \le 0, \qquad \dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\ \frac{\partial \tilde{G}}{\partial \boldsymbol{\sigma}} \tag{1.13}$$

with the standard conditions for $\dot{\lambda}$: $\dot{\lambda} \ge 0$, and $\dot{\lambda} = 0$ if $\tilde{F}(\boldsymbol{\sigma}) < 0$.
The criterion F and potential G are the Mohr-Coulomb or Drucker-Prager according to the 11th variable *Option*:

<u>**Option 0**: Mohr-Coulomb Criterion</u>

If *Option* = 0, F and G are the Mohr-Coulomb criterion and non-associate potential for the parameters C, ϕ , ψ and σ_T (see the model 31120).

$$F(\tilde{\boldsymbol{\sigma}}^p) = \frac{\tilde{\sigma}_1^p - \tilde{\sigma}_3^p}{2} + \frac{\tilde{\sigma}_1^p + \tilde{\sigma}_3^p}{2} \sin\phi - C\cos\phi \ \le\ 0 \tag{1.14}$$

$$G(\tilde{\boldsymbol{\sigma}}^p) = \frac{\tilde{\sigma}_1^p - \tilde{\sigma}_3^p}{2} + \frac{\tilde{\sigma}_1^p + \tilde{\sigma}_3^p}{2} \sin\psi \tag{1.15}$$

The Uniaxial Compressive Strength is then given by:

$$R_c(\overline{\theta}) = \frac{1}{\beta_{UCS}\left(\overline{\theta}\right)} \frac{2C\cos\phi}{1-\sin\phi} \tag{1.16}$$

Where:
If $f_T^2 > f_N$, or $b_T > 0$:

$$\beta_{UCS}\left(\overline{\theta}\right) = \frac{\sqrt{\left(1+a_N\sin^2\overline{\theta}\right)^2 + 4b_T\sin^2\overline{\theta}\cos^2\overline{\theta}} - \left(1+a_N\sin^2\overline{\theta}\right)\sin\phi}{1-\sin\phi} \tag{1.17}$$

If $f_T^2 < f_N$, or $b_T < 0$:

$$\beta_{UCS}\left(\overline{\theta}\right) = \frac{1}{2}\left(1+a_N\sin^2\overline{\theta} + \sqrt{\left(\left(1+a_N\sin^2\overline{\theta}\right)\right)^2 + 4b_T\sin^2\overline{\theta}\cos^2\overline{\theta}}\right) \tag{1.18}$$

For the special case $f_T^2 = f_N$, or $b_T = 0$, one finds:

$$b_T = 0 \ \rightarrow \qquad \beta_{UCS}\left(\overline{\theta}\right) = 1 + a_N\sin^2\overline{\theta} \tag{1.19}$$

This allows defining the adequate anisotropic UCS for a variety of rock-type materials. Two examples are given in the figures below for a rock with a weak anisotropy of UCS and a jointed rock with high UCS anisotropy.

Note that in all cases $\beta_{UCS}(0)=1,\;\; \beta_{UCS}(\pi/2)=f_N$ and this allows determining f_N or a_N. Then f_T or b_T can be determined or by considering the strength reduction in another direction, and instance in the direction θ=ω +π/4.

Example 1 : Rock with weak anisotropy
Consider a bedded or schistose rock with bedding plane making and angle ω with the x_1-*axis* of coordinates. Suppose that triaxial tests for compression axis parallel to the bedding plane have determined the cohesion *C* and friction angle ϕ so that the UCS in direction parallel to the bedding plane is $R_c(\omega)=\dfrac{2C\cos\phi}{1-\sin\phi}$. Then suppose that a *UCS* different with a factor 1/β is measured in the direction perpendicular to the bedding plane:

$$R_c(\omega+\frac{\pi}{2})=\frac{1}{\beta}R_c(\omega)$$

Then, we can take $a_N^p=\beta-1$ and $b_T^p=0$ to obtain an ellipsoidal shape of UCS in different directions for this rock (see Figure below left).

Example 2 : Jointed Rock
Consider a sedimentary or fractured rock mass with weakness planes making and angle ω with the x_1-*axis* of coordinates. Suppose that the strength criterion of the weakness planes or rock joints be given by a cohesion c^j and friction angle ϕ^j :

$$|\tau|=\sigma_n\tan\phi^j+c^j \tag{1.20}$$

Generally in this case the strength criterion of intact rock is assumed isotropic but in order to write a more general relation, we assume the UCS of the intact rock in the jointing direction is (1+a_N) time the strength in the perpendicular direction, with the possibility of taking a_N =0 for the isotropic intact rock matrix. The parameter b_T can be determined by considering the UCS in the direction making π/4 with the jointing plane. In this case we have $\overline{\theta}=\pi/4$ and on the joint plane we have:

$$\tau=\sigma\sin\overline{\theta}\cos\overline{\theta}=\sigma/2,\quad \sigma_n=\sigma\sin^2\overline{\theta}=\sigma/2$$

$$|\tau|=\sigma_n\tan\phi^j+c^j\quad\rightarrow\sigma=\frac{2c^j}{1-\tan\phi^j}$$

So the expected compressive strength for $\overline{\theta}=\pi/4$ is given by:

$$R_c(\pi/4)=\frac{2c^j}{1-\tan\phi^j} \tag{1.21}$$

Note that the expected $R_c(\pi/4)$ can result from theoretical calculation (1.21) or from experiment by testing samples oriented π/4 to the jointing plane. The compressive strength in the direction parallel to the jointing plane ($\overline{\theta}=0$) is that of the intact rock and given by

$$R_c(0)=\frac{2C\cos\phi}{1-\sin\phi} \tag{1.22}$$

Then we note:

$$\beta_{\pi/4}^{UCS}=\frac{R_c(0)}{R_c(\pi/4)}=\frac{2C\cos\phi/(1-\sin\phi)}{2c^j/(1-\tan\phi^j)} \tag{1.23}$$

The strength of the jointed rock in direction $\overline{\theta}=\pi/4$ is in principle smaller than that of the intact rock and so $\beta_{\pi/4}$ should be greater than 1 and then equation (1.17) with b_T >0 must be considered. For $\overline{\theta}=\pi/4$ this equation provides:

$$\beta_{UCS}(\pi/4)=\frac{\sqrt{(1+a_N/2)^2+b_T}-(1+a_N/2)\sin\phi}{1-\sin\phi} \tag{1.24}$$

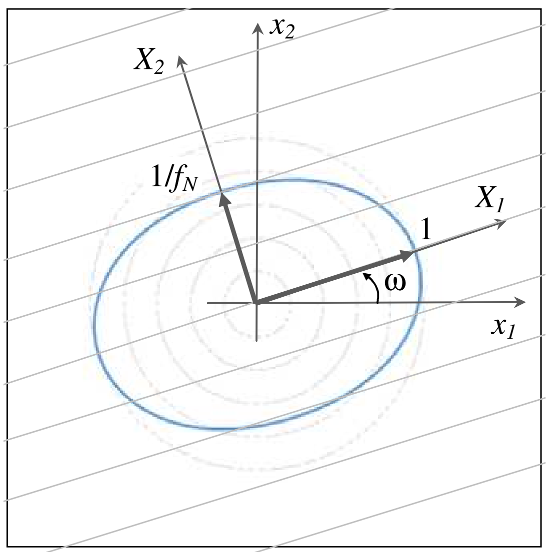
By solving the equation $\beta_{UCS}(\pi/4)=\beta_{\pi/4}^{UCS}$ one finds:

$$b_T=\left[\left(\beta_{\pi/4}^{UCS}\right)^2-(1+a_N/2)^2-\left(\beta_{\pi/4}^{UCS}-(1+a_N/2)\right)^2\sin\phi\right](1-\sin\phi) \tag{1.25}$$

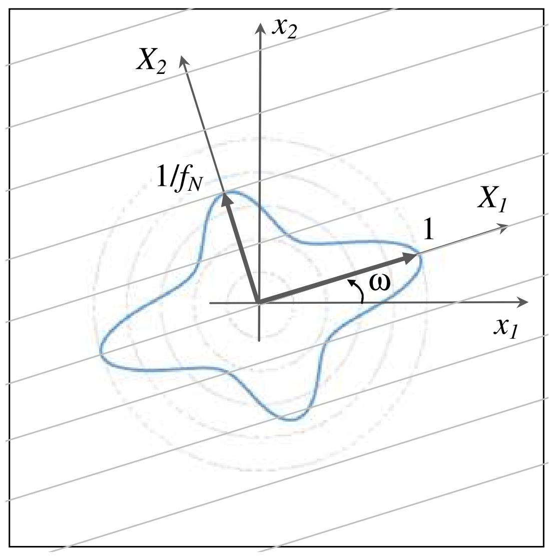
And for the simple case of isotropic intact rock (a_N=0, f_N=1):

$$b_T=\left[\left(\beta_{\pi/4}^{UCS}\right)^2-1-\left(\beta_{\pi/4}^{UCS}-1\right)^2\sin\phi\right](1-\sin\phi) \tag{1.26}$$

See an example of the UCS of this type of jointed rock in the figure below right.



(*a*)



(*b*)

Figure: Different cases of UCS anisotropy: (a) Weak anisotropy for rock matrix: ω=18°, $a_T^p=0.4$, $b_T^p=0$ and (b) anisotropic UCS for a jointed rock: ω=18°, $a_T^p=0.4$, $b_T^p=1.5$

Also the criterion is truncated by the traction limit σ_T (see the material 31120). Note the transformed stress $\tilde{\sigma}^p$ will be compared to σ_T and so the tensile strength will be anisotropic in the same way that the plastic criterion.

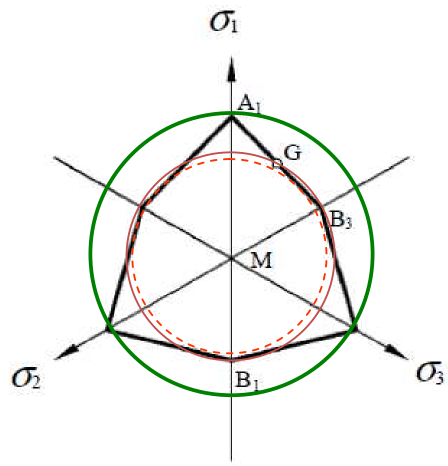
Options 1,2,3 : Drucker-Prager Criterion

If *Option* = 1,2,3 *F* and *G* are the Drucker-Prager criterion and non-associate potential for the parameters *K*, α_ϕ and α_ψ defined as follows:

Option =1: External corners: $K=\dfrac{6C\cos\phi}{3-\sin\phi}$, $\sin\alpha_\phi=\dfrac{2\sin\phi}{3-\sin\phi}$, $\sin\alpha_\psi=\dfrac{2\sin\psi}{3-\sin\psi}$ (1.27)

Option =2: Internal corners: $K=\dfrac{6C\cos\phi}{3+\sin\phi}$, $\sin\alpha_\phi=\dfrac{2\sin\phi}{3+\sin\phi}$, $\sin\alpha_\psi=\dfrac{2\sin\psi}{3+\sin\psi}$ (1.28)

Option =3: Tangent to faces: $K=\dfrac{3C\cos\phi}{\sqrt{3+\sin^2\phi}}$, $\sin\alpha_\phi=\dfrac{\sin\phi}{\sqrt{3+\sin^2\phi}}$, $\sin\alpha_\psi=\dfrac{\sin\psi}{\sqrt{3+\sin^2\psi}}$ (1.29)



Equivalent Drucker-Prager parameters for Mohr-Coulomb: Option 1: circle passing by external corners (green circle), Option 2: passing by internal corners (red circle), Option 3: tangent to faces (dashed-line circle).

F is Drucker-Prager criterion calculated with the transformed stress $\tilde{\sigma}_e^p$ and the parameters K, α_ϕ and $\tilde{G}$ is the plastic potential with a different dilatancy angle α_ψ :

$$\tilde{F}(\boldsymbol{\sigma})=F(\tilde{\boldsymbol{\sigma}}^p)=\tilde{\sigma}_e^p+\sin\alpha_\phi\,\tilde{I}^p-K \quad , \qquad \tilde{G}(\boldsymbol{\sigma})=G(\tilde{\boldsymbol{\sigma}}^p)=\tilde{\sigma}_e^p+\sin\alpha_\psi\,\tilde{I}^p \quad (1.30)$$

$\tilde{\sigma}_e^p$ and $\tilde{I}^p$ are the equivalent stress and the first invariant associated to $\tilde{\boldsymbol{\sigma}}^p$:

For Drucker-Prager case (options 1,2,3), the tensile strength truncation σ_T will not be taken into account.

Option 4 : Plane Mohr-Coulomb Criterion

If *Option* = 4 F and G are the Plane Mohr-Coulomb criterion and non-associate potential for the parameters C, ϕ , ψ and σ_T (see the model 31120). In Plane Mohr-Coulomb (PMC) criterion, the out-of-plane stress is not considered or, equivalently, is supposed to be the intermediate principal stress. The extreme principal stresses are deduced from the stress components (σ_{xx}, σ_{yy}, σ_{xy}):

$$\sigma_1=\frac{\sigma_{xx}+\sigma_{yy}+\sqrt{\left(\sigma_{xx}-\sigma_{yy}\right)^2+4\sigma_{xy}^2}}{2}$$
$$\sigma_3=\frac{\sigma_{xx}+\sigma_{yy}-\sqrt{\left(\sigma_{xx}-\sigma_{yy}\right)^2+4\sigma_{xy}^2}}{2} \quad (1.31)$$

And with these stresses, calculated from the transformed stress tensor, the criterion F and plastic potential G are the same that (1.14) and (1.15) for the Mohr-Coulomb option.

The tensile strength truncation σ_T is taken into account in this PMC material.

Softening plasticity

The cohesion C, the tensile strength σ_T and the angle ϕ in the here-above relations can vary with the plastic shear strain γ. The evolution is given by the *hardening law* (including softening as negative hardening) depending on three additional parameters: the *residual cohesion* C_r, the *residual friction angle* ϕ_r and the brittleness parameter B. Two parameters of *cohesion and tensile strength reduction* η_c and *friction angle reduction* η_ϕ are defined as follows:

$$\eta_c = 1 - \frac{C_r}{C_i} = 1 - \frac{\sigma_T}{\sigma_{Ti}}, \quad \eta_\phi = 1 - \frac{\tan\phi_r}{\tan\phi_i} \tag{1.32}$$

C_i is the initial or intact cohesion, σ_{Ti} the initial tensile strength and ϕ_i the initial friction angle which are constant parameters of the material. For simplicity of notation, they are designated by C, ϕ and σ_T in the list of parameters below (Param7, Param8 and Param10).

The *cumulated plastic strain* γ includes contributions from the plastic shear strain and from the plastic extension. Irreversible shear can degrade the cohesion of the material. Positive values of diagonal components of the plastic strain, representing extensional deformation created by tensile stresses, can also contribute to decohesion of the material. So γ includes two types of contributions and it affects also the cohesion C of the material as well as it tensile strength σ_T. It is calculated in the following way:

$$\dot{\gamma} = \frac{\dot{\gamma}_s + \dot{\gamma}_T}{2} \tag{1.33}$$

The shear contribution part $\dot{\gamma}_s$ is calculated from the deviatoric plastic strain increment $\dot{\boldsymbol{e}}^p$ by the following relations:

$$\dot{\gamma}_s = \sqrt{\frac{2}{3}\dot{\boldsymbol{e}}^p : \dot{\boldsymbol{e}}^p} \qquad \dot{\boldsymbol{e}}^p = \dot{\boldsymbol{\varepsilon}}^p - \frac{1}{3}\dot{\varepsilon}_v^p\,\boldsymbol{\delta}, \quad \dot{\varepsilon}_v^p = \dot{\boldsymbol{\varepsilon}}^p : \boldsymbol{\delta} \tag{1.34}$$

The traction part $\dot{\gamma}_T$ is calculated from the positive eigenvalues of the plastic strain rate tensor. The three eigenvalues are $\dot{\varepsilon}_{33}^p$ and the two in-plane values:

$$\dot{\varepsilon}_+^p = \frac{\dot{\varepsilon}_{11}^p + \dot{\varepsilon}_{22}^p + \sqrt{\left(\dot{\varepsilon}_{11}^p - \dot{\varepsilon}_{22}^p\right)^2 + 4\left(\dot{\varepsilon}_{12}^p\right)^2}}{2}, \quad \dot{\varepsilon}_-^p = \frac{\dot{\varepsilon}_{11}^p + \dot{\varepsilon}_{22}^p + \sqrt{\left(\dot{\varepsilon}_{11}^p - \dot{\varepsilon}_{22}^p\right)^2 + 4\left(\dot{\varepsilon}_{12}^p\right)^2}}{2} \tag{1.35}$$

And $\dot{\gamma}_T$ is the sum of the positive values of these eigenstrains:

$$\dot{\gamma}_T = \frac{\dot{\varepsilon}_+^p + \left|\dot{\varepsilon}_+^p\right|}{2} + \frac{\dot{\varepsilon}_-^p + \left|\dot{\varepsilon}_-^p\right|}{2} + \frac{\dot{\varepsilon}_{33}^p + \left|\dot{\varepsilon}_{33}^p\right|}{2} \tag{1.36}$$

It can be noted that for a:

$$\text{Simple shear: } \dot{\boldsymbol{\varepsilon}}^p = \begin{bmatrix} 0 & \dot{\varepsilon}_{12}^p & 0 \\ \dot{\varepsilon}_{12}^p & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \dot{\gamma}_s = \frac{2}{\sqrt{3}}\left|\dot{\varepsilon}_{12}^p\right|, \ \dot{\gamma}_T = \left|\dot{\varepsilon}_{12}^p\right|, \ \rightarrow \ \dot{\gamma} = \left(\frac{1}{2} + \frac{1}{\sqrt{3}}\right)\left|\dot{\varepsilon}_{12}^p\right| \tag{1.37}$$

And if the plastic strain is traceless ($tr\dot{\boldsymbol{\varepsilon}}^p = 0$) then for uniaxial traction or compression:

$$\dot{\boldsymbol{\varepsilon}}^{p}=\begin{bmatrix}\dot{\varepsilon}_{11}^{p} & 0 & 0\\ 0 & -\dot{\varepsilon}_{11}^{p}/2 & 0\\ 0 & 0 & -\dot{\varepsilon}_{11}^{p}/2\end{bmatrix} \tag{1.38}$$

Then for:

Simple traction: $\dot{\varepsilon}_{11}^{p}>0\rightarrow\quad \dot{\gamma}_{s}=\dot{\varepsilon}_{11}^{p},\quad \dot{\gamma}_{T}=\dot{\varepsilon}_{11}^{p}\quad\rightarrow\quad \dot{\gamma}=\dot{\varepsilon}_{11}^{p}$ (1.39)

Simple compression: $\dot{\varepsilon}_{11}^{p}<0\rightarrow\quad \dot{\gamma}_{s}=\left|\dot{\varepsilon}_{11}^{p}\right|,\quad \dot{\gamma}_{T}=\left|\dot{\varepsilon}_{11}^{p}\right|\quad\rightarrow\quad \dot{\gamma}=\left|\dot{\varepsilon}_{11}^{p}\right|$ (1.40)

The evolution of C, ϕ and the tensile strength σ_T in ANELVIP is calculated in a general way by:

$$C(\gamma)=\left(1-V_{c}\right)C_{i}\ ,\qquad \sigma_{T}(\gamma)=\left(1-V_{T}\right)\sigma_{Ti}\ ,\qquad \tan\phi(\gamma)=\left(1-V_{\phi}\right)\tan\phi_{i} \tag{1.41}$$

where V_c , V_T and V_ϕ are internal variables of the material. Theses internal variables are calculated from the *cumulated plastic strain* γ by the following relations:

$$V_{c}(\gamma)=\eta_{c}\left(1-e^{-B\gamma}-M\gamma e^{-B\gamma}\right),\qquad V_{T}(\gamma)=\eta_{c}\left(1-e^{-B\gamma}\right),\qquad V_{\phi}(\gamma)=\eta_{\phi}\left(1-e^{-B\gamma}\right) \tag{1.42}$$

B is a positive parameter characterizing the brittleness of the material: the decrease of the strength parameters C, ϕ and σ_T is faster for greater B. The friction angle and the tensile strength can only decrease whereas the cohesion evolution depending on the parameter M, and so the compression curve, can present a positive hardening phase and a peak value.

If $M < B$, the cohesion $C(\gamma)$ is always decreasing, but if $M > B$ then $C(\gamma)$ starts by increasing and attains, for a cumulated shear denoted by γ_{peak} a maximum value denoted by C_{peak}. These values can be determined by derivation of the first relation in (1.42) and one finds:

$$\gamma_{peak}=\frac{1-g}{gM}\,,\qquad C_{peak}=\left[1-\eta_{c}+\frac{\eta_{c}}{ge^{1-g}}\right]C_{i} \tag{1.43}$$

where:

$$g=\frac{B}{M} \tag{1.44}$$

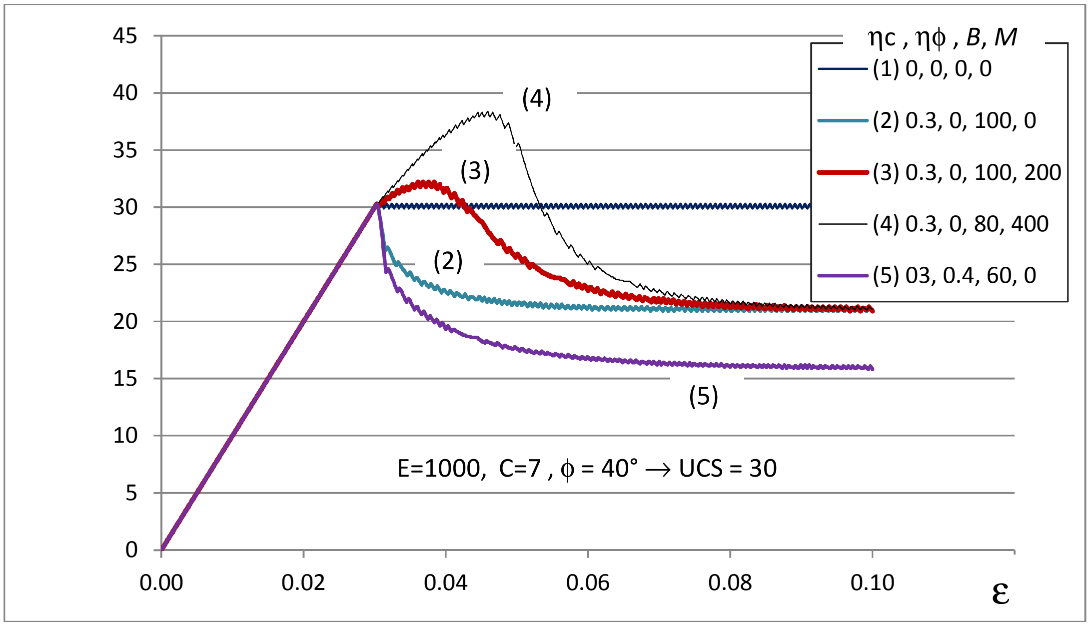


Figure: Stress-Strain curves for a uniaxial compression test on Anelvip elastoplastic material with different softening parameters.

The perfect plastic material is obtained by posing $\eta_c=\eta_\phi=0$. In this case no evolution is calculated for C, ϕ and σ_T and B is not used.

Note: The softening behavior leads to localization and mechanical instabilities which can well be modeled in Disroc with this Anelvip model. The localization in a sample affects its nominal stress-strain curve. The curves in the figure above are obtained on a FEM model with *one only* (quadrilateral) element in order to avoid localization effects.

Determination of softening parameters

The two equations (1.43) and (1.44) allow determining the two parameters B and M from γ_{peak} and C_{peak} values given by the experimental curves.

However, different methods can be used to determine these two parameters depending on which aspect of experimental curves is more important to reproduce more accurately.

A first method could be to determine B from the variation of C if pure shear test data are available or from the variation of σ_T if simple traction curves are available. This can happen if numerical homogenization test data are being analyzed. After B is determined it is easier to determine M from (1.43), (1.44) and γ_{peak} value (see below for estimation of γ_{peak}).

If only simple compression test data are considered for determination of B and M then different methods can be used. For instance, let σ_i designate the elastic stress limit (the end of the elastic stage) and σ_{peak} for the maximum stress (Figure) and suppose that the friction angle remains constant ($\eta_\phi=0$). From the relation between the UCS and the cohesion, $R_c=2C\cos\phi/(1-\sin\phi)$, one finds:

$$\frac{C_{peak}}{C_i} = \frac{\sigma_{peak}}{\sigma_i} \qquad (1.45)$$

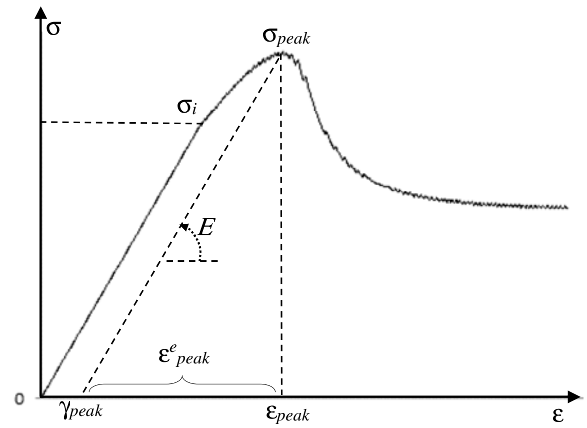


Figure: Determination of γ_{peak} and σ_{peak} from experimental curves

Also, if the axial strain at the peak stress is ε_{peak} then:

$$\gamma_{peak} = \varepsilon_{peak} - \sigma_{peak}/ E \qquad (1.46)$$

Note that this relation is valid only for a uniaxial compression with monotonic loading and with *E* the Young's modulus in the compression direction. In addition, this relation supposes a constant friction angle and also the expression (1.38) of the traceless plastic strain. These assumptions are not always satisfied and specially the last one, (1.38), is not true for Mohr-Coulomb and Drucker-Parger criteria with associate flow rule. In these cases, the equations (1.45) and (1.46) must be considered as approximate relations allowing to determine a first trial set of values for *B* and *M* and then determine more accurate values for these parameters by numerical simulation of theoretical curves and comparison to the experimental ones.

From the equation (1.43) one can deduce:

$$\frac{\eta_c C_i}{C_{peak} - C_i + \eta_c C_i} = g e^{1-g} \qquad (1.47)$$

The value of the expression at the left side of (1.47) can be determined from experimental data. But this equation can not be solved explicitly to determine *g*. The following figure allows finding *g* from the left-side value of (1.47).

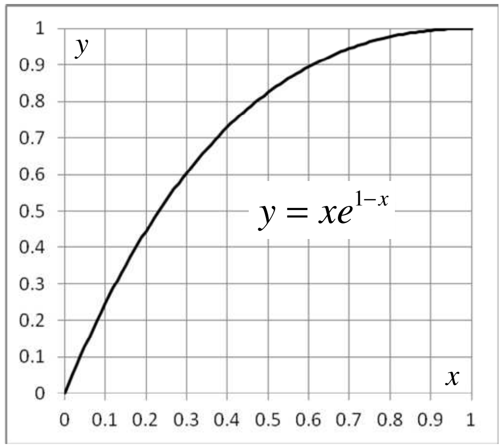


Figure: The function allowing to determine g

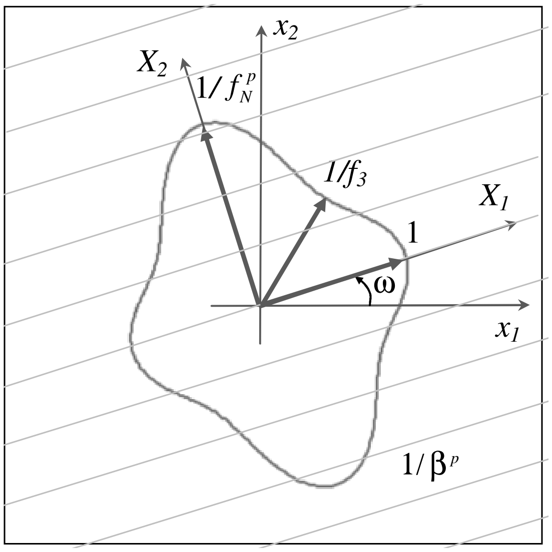
Once g has been determined, B and M can be determined from γ_{peak} by:

$$B = \frac{1-g}{\gamma_{peak}} \quad , \qquad M = \frac{1-g}{g\ \gamma_{peak}} \tag{1.48}$$

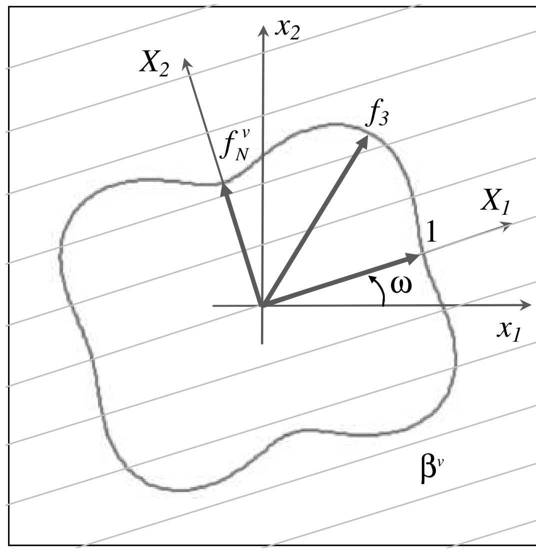
However, as mentioned here above the determination of γ_{peak} is not easy in the general case.
It can be determined by an iterative method:
First, with starting with the value given by (1.46), a first estimate for B and M is determined by (1.48). Then the theoretical curve obtained by these parameters is compared to the experimental one. The ε_{peak} is easily determined from the experimental curve. γ_{peak} and ε_{peak} vary in the same way. So if the theoretical ε_{peak} is smaller than the experimental one, a greater value for γ_{peak} is adopted to determine new values for B and M. The process is repeated until sufficiently precise values are determined for these parameters.

II) Viscous deformation

An anisotropic extension of the isotropic creep law can be defined by making anisotropic both the strain rate and the stress threshold with two different sets of a_N and a_T denoted by $(a_N^v\ ,\ a_T^v)$ for viscous set and $(a_N^p\ ,\ a_T^p)$ for plastic model or stress threshold: viscous strain rate will be multiplied by β^v and the stress threshold divided by β^p (Figure).



Indicator surface of $1/\beta^p$



Indicator surface of β^v

If the uniaxial stress σ_θ is applied in a direction making an angle θ with respect X_1 then the axial creep strain ε_θ measured in this direction is assumed to be:

$$\varepsilon_\theta(t) = a\,\beta^v(\theta) < \beta^p(\theta)\sigma_\theta - \sigma_c >^n t^\alpha \qquad (1.49)$$

where a, n, α, σ_c are four material constants, $\beta^v(\theta)$ and $\beta^p(\theta)$ two direction dependency coefficients for the stain rate and the stress threshold and the *positive part* function $<\cdot>$ is defined as:

$$\begin{aligned} < x > &= 0 \quad \textit{if} \quad x < 0 \\ < x > &= x \quad \textit{if} \quad x \geq 0 \end{aligned}$$

The four parameters a, n, α, σ_c can be identified from uniaxial creep results If α =1, the Norton-Hoff creep model is recovered.

The incremental constitutive equation for creep function (1.49) is written by introducing the auxiliary parameter ξ and the transformed stresses $\tilde{\sigma}_e^p$, $\tilde{\sigma}_e^v$, $\tilde{\boldsymbol{S}}^v$ with:

$$\dot{\xi} = \left(a\,\beta^v < \tilde{\sigma}_e^p - \sigma_c >^n \right)^{1/\alpha} \qquad (1.50)$$

And

$$\dot{\boldsymbol{\varepsilon}}^v = \frac{3}{2}\alpha\,\xi^{\alpha-1}\dot{\xi}\,\frac{\tilde{\boldsymbol{S}}^v}{\tilde{\sigma}_e^v} \qquad (1.51)$$

To avoid numerical problems near $\xi = 0$, the law is completed by:

$$\dot{\boldsymbol{\varepsilon}}^v = \frac{3}{2}\alpha\,\varepsilon_0^{\alpha-1}\dot{\xi}\,\frac{\tilde{\boldsymbol{S}}^v}{\tilde{\sigma}_e^v} \quad \text{if} \quad \xi^\alpha \leq \varepsilon_0 \qquad (1.52)$$

Thus an additional parameter ε_0 is introduced. The transformed stresses $\tilde{\boldsymbol{\sigma}}^v, \tilde{\boldsymbol{S}}^v, \tilde{\sigma}_e^v$ are defined by transformation with $(a_N\,,\ a_T) = (a_N^v\,,\ a_T^v)$. The viscosity anisotropy parameter β is defined by the same expression (1.9),(1.10) but with parameters $(a_N^v\,,\ a_T^v)$:

$$\beta^v = \frac{\tilde{\sigma}_e^v}{\sigma_e} \tag{1.53}$$

The transformed stresses $\tilde{\boldsymbol{\sigma}}^p$, $\tilde{\boldsymbol{S}}^p$, $\tilde{\sigma}_e^p$ are defined by transformation with $(a_N\ ,\ a_T) = (a_N^p\ ,\ a_T^p)$
Thus, the anisotropy is defined by two sets of parameters $(a_N^v\ ,\ a_T^v)$ and. $(a_N^p\ ,\ a_T^p)$.

Note that if the stress threshold σ_c is greater than plastic strength then no viscous strain will be produced because the stress remaining in the elastic domain defined by the plastic criterion cannot exceed σ_c.

Nb = 24
Param1 = E_1
Param2 = E_2
Param3 = ν_{12}
Param4 = ν_{13}
Param5 = μ_{12}
Param6 = ω (in degrees)
Param7 = C (C_i if evolution)
Param8 = ϕ (in degrees)
Param9 = ψ (in degrees)
Param10 = σ_T
Param11 = Mohr-Coulomb/Drucker-Prager Option) (MC:0, DP*e*:1, DP*i*:2, DP*f*:3, PMC:4)
Param12 = $a^p{}_N$
Param13 = $b^p{}_T$
Param14 = a (attention to the stress and time units)
Param15 = n
Param16 = α
Param17 = σ_c
Param18 = $a^v{}_N$
Param19 = $b^v{}_T$
Param20 = ε_0
Param21 = η_c (*cohesion reduction*)
Param22 = η_ϕ (*friction angle reduction*)
Param23 = B (*plasticity brittleness*)
Param24 = M (*positive hardening parameter*)

Note

- If $C \geq 10E_1$ no plastic strain will be calculated (the model becomes viscoelastic). The parameters 6, 7 and 11 have no effects. But $a^p{}_N$ and $a^p{}_T$ can be used for viscous strain.
- If a =0, no viscous strain will be calculated (the model becomes elastoplastic). The parameters 13 to 18 will not be used.

- If η_c=η_ϕ=0 no hardening or softening evolution for C et ϕ and B is not used.

Internal Variables:
Vin(n,1): reserved for damage (not existing for this material)
Vin(n,2) : ξ, internal

Vin(n,3): Plastic shear deformation γ
Vin(n,4) : Reduction factor for cohesion, V_c
Vin(n,5) : Reduction factor for friction angle, V_ϕ
Vin(n,6) : Reduction factor for tensile strength, V_T

31430	ANELVIP: Anisotropic ElastoViscoPlasticity Mohr-Coul.(0)/Druck.-Prag.(1,2,3)+ Creep
Nb: 24	
Param1 = E_1	Param13 = $b^p{}_T$
Param2 = E_2	Param14 = a
Param3 = ν_{12}	Param15 = n
Param4 = ν_{13}	Param16 = α
Param5 = μ_{12}	Param17 = σ_c
Param6 = ω (in degrees)	Param18 = $a^v{}_N$
Param7 = C	Param19 = $b^v{}_T$
Param8 = ϕ (in degrees)	Param20 = ε_0
Param9 = ψ (in degrees)	Param21 = η_c
Param10 = σ_T	Param22 = η_ϕ
Param11 =*Option* (01,2,3,4)	Param23 = B
Param12 = $a^p{}_N$	Param24 = M

31600 : Elastic-Damage material with modified Drucker-Prager softening criterion

Note: Model to be developed. Not available!
Isotropic elasticity with damage:

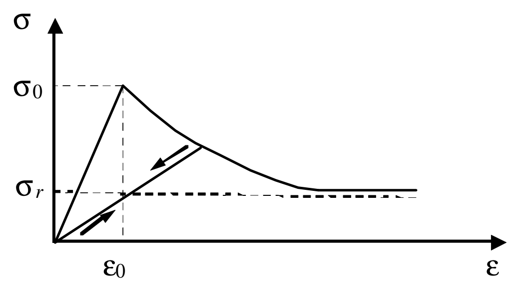
$$\boldsymbol{\varepsilon} = \frac{1+\nu}{E(1-D)}\boldsymbol{\sigma} - \frac{\nu}{E(1-D)} tr(\boldsymbol{\sigma})\,\boldsymbol{\delta}$$

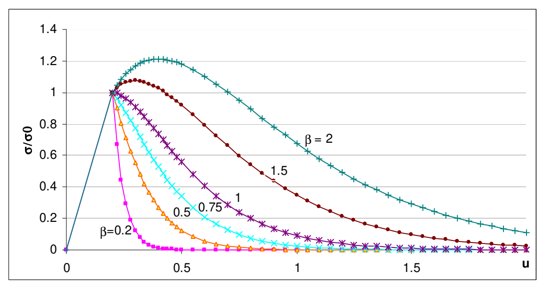
Damage criterion:

$$F(\boldsymbol{\sigma}, D) = \sqrt{\sigma_e^2 + b^2 g^2} + \sin\alpha\, I_1 - gK$$

$$\sigma_e = \sqrt{3J_2}, \quad I_1 = tr(\boldsymbol{\sigma})$$

$$g(D) = \eta_r + (1-\eta_r)(1-D)\left[1-\beta \ln(1-D)\right], \qquad \eta_r = \frac{\sigma_r}{\sigma_0}$$



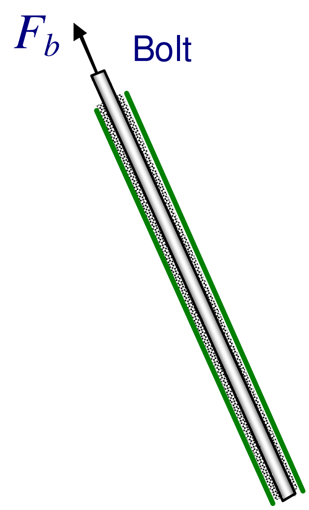


Nb = 7
Param1 = E
Param2 = ν
Param3 = sin α
Param4 = K
Param5 = β
Param6 = η_r
Param7 = b
Variable interne
Vin(n,1) : D
Condition : $b \cos\alpha < K$ must be satisfied.

31600	Elastic-Damage material with modified Drucker-Prager softening criterion
Nb: 7	
Param1 = E	
Param2 = ν	
Param3 = sin α	
Param4 = K	
Param5 = β	
Param6 = η_r	*Condition* : $b \cos\alpha < K$
Param7 = b	

I.4) Mechanics - ANCHORS

41100 : Elastic Rock Anchor



Axial deformation of the anchor rod: $\varepsilon = \dfrac{F_b - F_0}{ES}$

F_b axial force in the rod,
F_0 prestress axial force,

Elastic contact between rod and rock : $\underline{\sigma} = \boldsymbol{K}\,\underline{u}$
(the same model 21100)

Note: For the section S to take into account the same remarks that for bar elements (material model 11100) are valid. The stiffness parameters K_t, K_n and K_{tn} here take into account the circumference of the steel rod as well as the number of anchors per unit thickness of the model. For instance, if the grout filling the space between the rod and the rock has a thickness e and a shear modulus μ, then it correspond to a physical stiffness μ/e (see the material 21100). Then if the rod has a diameter D then the Param2 = $K_t = \pi D\ \mu/e$. In addition, if in the unit thickness of the plane of the model there are n anchors (see the note for the bar elements 11100), then Param2 = $K_t = n\ \pi D\ \mu/e$. The same method is to be applied to K_t and K_{tn}.

Nb = 5
Param1 = ES (Young's modulus (steel) × section)
Param2 = K_t (tangent stiffness)
Param3 = K_n (normal stiffness)
Param4 = $K_{nt} = K_{tn}$ (non diagonal stiffness term causing dilatancy)
Param5 = F_0 (prestress force)

41100	Elastic Anchor
Nb: 5	
Param1 = ES (Young's modulus (steel) × section)	
Param2 = K_t (tangent stiffness)	
Param3 = K_n (normal stiffness)	
Param4 = $K_{nt} = K_{tn}$ (non diagonal stiffness term causing dilatancy)	
Param5 = F_0 (prestress force)	

41110 : Elastic-Plastic Rock Anchor

Axial deformation of the anchor rod: $\varepsilon - \varepsilon^p = \dfrac{F_b - F_0}{ES}$

In monotonic loading $\varepsilon^p < 0$ if $F_b < Y_s$ where:
$Y_s = \sigma_y S$ with σ_y the plastic limit stress of the rod (steel) and S the rod section

Contact between rod and rock : $\underline{\sigma} = \boldsymbol{K}\,(\underline{u} - \underline{u}^p)$
Plastic criterion for rod-rock contact: $f(\underline{\sigma}) = |\tau| + \sigma_n \tan\phi - c \leq 0$

Contact model: the same that the model 21120

Nb = 8
Param1 = ES (Young's modulus (steel) × section)
Param2 = K_t (tangent stiffness)
Param3 = K_n (normal stiffness)
Param4 = $K_{nt} = K_{tn}$ (non diagonal stiffness term causing dilatancy)
Param5 = Y_s (plastic limit for the axial force in the anchor)
Param6 = C (cohesion)
Param7 = ϕ (in degrees, the friction angle)
Param8 = F_0 (prestress force)

Note: The method of calculation of S, K_t, K_n, K_{tn} and Y_s is the same that for materials 41100 et 11110. The cohesion parameter C is the product of the physical cohesion of the contact between the rod and the rock (cohesion of the grout material) and the circumference of the rod, and also the number of anchors per unit thickness of the plane model (see materials 41100 and 11100). The angle ϕ is the friction angle (in degrees) of the contact (or the grout material).

41110 Elastic-Plastic Anchor

Nb: 8
Param1 = ES (Young's modulus (steel) × section)
Param2 = K_t (tangent stiffness)
Param3 = K_n (normal stiffness)
Param4 = $K_{nt} = K_{tn}$ (non diagonal stiffness → dilatancy)
Param5 = Y_s (plastic limit for axial *force* in the anchor)
Param6 = C (cohesion)
Param7 = ϕ (in degrees, the friction angle)
Param8 = F_0 (prestress force)

41310 : Elastic-Damage Rock Anchor

Axial deformation of the anchor rod: $\varepsilon - \varepsilon^p = \dfrac{F_b - F_0}{ES}$

In monotonic loading $\varepsilon^p < 0$ if $F_b < Y_s$ where:
$Y_s = \sigma_y S$ with σ_y the plastic limit stress of the rod (steel) and S the rod section
Contact between rod and rock :

$$\underline{\sigma} = (\boldsymbol{K}_D + \boldsymbol{k}_r)\,(\underline{u} - \underline{u}^p)$$

With:

$$\boldsymbol{K}_D = \begin{bmatrix} (1-D)K_t & 0 \\ 0 & K_n \end{bmatrix}, \qquad \boldsymbol{k}_r = \begin{bmatrix} k_{rt} & 0 \\ 0 & 0 \end{bmatrix} \quad \underline{u}^p = \begin{bmatrix} u_t^p \\ 0 \end{bmatrix}$$

Damage criterion for rod-rock contact: $F(\underline{\sigma}, D) = |\tau| - g(D)\,C$

With: $g(D) = (1-D)\big(1 - \beta\, ln(1-D)\big)$

The plastic deformation of the contact takes place after the complete damage. The plastic criterion is:

$$F^p(\underline{\sigma}) = |\tau| - C_r$$

Note: The strength parameter takes into account, in the same way that *C*, the circumference of the rod and the number of anchors per unit thickness of the model (see the model 41110).

Nb = 9
Param1 = *ES* (Young's modulus (steel) × section)
Param2 = K_t (tangent stiffness)
Param3 = K_n (normal stiffness)
Param4 = Y_s (plastic limit for the axial force in the rod)
Param5 = *C* (cohesion)
Param6 = C_r (residual cohesion)
Param7 = β (ductility)
Param8 = k_{rt} (residual tangent stiffness)
Param9 = *Option* (1 if plasticity taken into account)

Internal variable
Vin(n,1) : *D*

41310 Elastic-Damage Anchor

Nb = 9
Param1 = *ES* (Young's modulus (steel) × section)
Param2 = K_t (tangent stiffness)
Param3 = K_n (normal stiffness)
Param4 = Y_s (plastic limit for the axial force in the rod)
Param5 = *C* (cohesion)
Param6 = C_r (residual cohesion)
Param7 = β (ductility)
Param8 = k_{rt} (residual tangent stiffness)
Param9 = *Option* (1 if plasticity taken into account)

51100 : Elastic Beam

The efforts in elastic beam are the axial force F, the shear force V and the bending moment M. They are related to the axial strain ε and the rotation θ by:

$$F = ES\,\varepsilon \;\;,\quad M = EI\,\frac{d\theta}{ds}$$

The shear force is related to M by $V = -dM/dx$ where x designates the position x along the beam.

Nb = 2
Param1 = ES : Young's modulus (steel) × S (section)
Param2 = EI : Young's modulus (steel) × I (moment of inertia)

Note: The 2D plane modeling, a unit the thickness of the model is considered in relation with a 3D modeling. The section S and the inertia moment I are supposed to correspond to a unit thickness of the model. If there are more or less one beam per unit thickness, these parameters must be multiplied by the number of beams by unit thickness (see for the bar element 11100).

51100	Elastic Beam
Nb: 2	
Param1 = ES	: Young's modulus (steel) × S (section)
Param2 = EI	: Young's modulus (steel) × I (moment of inertia)

61100 : Elastic Bolt (beam + contact interface)

Bolt is anchor element with bending and shear effects for the steel rod. The steel rod is modeled as beam element and the contact between the rod and the rock, modeled by a joint element.

The efforts in elastic beam are the axial force F, the shear force V and the bending moment M. They are related to the axial strain ε and the rotation θ by:

$$F = ES\,\varepsilon \ , \quad M = EI\,\frac{d\theta}{ds}$$

The shear force is related to M by $V = -dM/dx$ where x designates the position x along the beam (the same model 51100).

Elastic contact between rod and rock : $\underline{\sigma} = \boldsymbol{K}\,\underline{u}$
(the same model 21100, 41100)

Nb = 5
Param1 = ES Young's modulus (steel) × section
Param2 = EI : Young's modulus (steel) ×
Param3 = K_t (tangent stiffness)
Param4 = K_n (normal stiffness)
Param5 = $K_{nt} = K_{tn}$ (non diagonal stiffness term causing dilatancy)

Note: For the section S and the moment of iniertia to take into account the same remarks that for bar elements.

61100	Elastic Bolt (beam + contact interface)
Nb: 5	
Param1 = ES	Young's modulus (steel) × section
Param2 = EI	: Young's modulus (steel) × intertia
Param3 = K_t	(tangent stiffness)
Param4 = K_n	(normal stiffness)
Param5 = $K_{nt} = K_{tn}$	(non diagonal stiffness → dilatancy

61110 : Elastic Bolt with elastoplastic contact

Bolt is anchor element with bending and shear effects for the steel rod. The steel rod is modeled as beam element and the contact between the rod and the rock, modeled by a Mohr-Coulomb elastoplastic contact interface. This model is the extension of the model 61100 to the plasticity of the interface or of the cable model 41110 to accounting for bending moment but without plasticity of the steel rod and without pre-stress .

The efforts in elastic beam are the axial force F, the shear force V and the bending moment M. They are related to the axial strain ε and the rotation θ by:

$$F = ES\,\varepsilon \;\;,\quad M = EI\,\frac{d\theta}{ds}$$

The shear force is related to M by $V = -dM/dx$ where x designates the position x along the beam (the same model 51100).

Contact between rod and rock : $\underline{\sigma} = \boldsymbol{K}\,(\underline{u} - \underline{u}^{p})$
Plastic criterion for rod-rock contact: $f(\underline{\sigma}) = |\tau| + \sigma_n \tan\phi - c \le 0$
(the same model 21120, 41110)

Nb = 5
Param1 = ES Young's modulus (steel) × section
Param2 = EI : Young's modulus (steel) ×
Param3 = K_t (tangent stiffness)
Param4 = K_n (normal stiffness)
Param5 = $K_{nt} = K_{tn}$ (non diagonal stiffness term causing dilatancy)
Param6 = C (cohesion of the steel-rock contact)
Param7 = ϕ (in degrees, the friction angle of the contact)

Note: For the section S and the moment of inertia to take into account see the same remarks that for bar elements. For the paremeters K_t, K_n, K_{tn} and the cohesion C see the same remark that for the material 41110.

61110 Elastic Bolt (beam + contact interface)

Nb: 7
Param1 = ES Young's modulus (steel) × section
Param2 = EI : Young's modulus (steel) × inertia
Param3 = K_t (tangent stiffness)
Param4 = K_n (normal stiffness)
Param5 = $K_{nt} = K_{tn}$ (non diagonal stiffness → dilatancy
Param6 = C (cohesion)
Param7 = ϕ (in degrees, the friction angle)

II) Hydraulic

II.1) Hydraulic - BOREHOLES & TUBES
(associated hydraulic model for bars, beams, anchors and bolts)

12100 : Borehole : Steady state flow

The pressure in the borehole is the same that at its wall for the surrounding porous matrix. This model is suitable for calculating steady state flow.

Constitutive law: $q = -C_t \nabla p$

q : debit in the tube, ∇p : fluid pressure gradient along the tube line

Nb = 1

Param1 = C_t (tangent or longitudinal conductivity)

Note: q is the integral of the fluid velocity in the section of the tube ($q = ve$).
Tube elements are the hydraulic model associated to bar elements (Mechanics). If bar elements are present in the mechanical model, they will be present also in the hydraulic mesh and their hydraulic model must be specified. Put $C_t = 0$ if they have no contribution to hydraulic flow.

12100	Borehole: Hydraulic model steady state
Nb: 1	
Param1 = C_t (tangent or longitudinal conductivity)	

12110 : Borehole : Transient flow

The pressure in the borehole is the same that at its wall for the surrounding porous matrix. This model allows calculating transient flow.

Constitutive law: $q = -C_t \nabla p$, $C_M \dfrac{\partial p}{\partial t} = \nabla.(C_t \nabla p)$

q : debit in the tube, ∇p : fluid pressure gradient along the tube line

Nb = 2

Param1 = C_t (tangent or longitudinal conductivity)

Param2 = C_M (storage coefficient)

Note: See the note for the material 12100

12110	Borehole: transient flow
Nb: 2	
Param1 = C_t (tangent or longitudinal conductivity)	
Param2 = C_M (storage coefficient)	

12200 : Tube : Steady state flow

The pressure inside the tube is different from the pressure on its outside wall for the surrounding porous matrix. This model is suitable for calculating steady state flow.

Constitutive law: $q = -C_t \nabla P$, $\nabla.(C_t \nabla P) + C_n (p - P) = 0$

P : pressure inside the tube which can be different from the outside pressure
∇P : fluid pressure gradient along the tube line
p : pressure outside the tube
q : debit in the tube, ∇p : fluid pressure gradient along the tube line

Nb = 2
Param1 = C_t (tube longitudinal conductivity)
Param2 = C_n (wall-through conductivity, zero if impervious wall)

Note: For this model the pressure is continuous in the matrix when crossing the tube but different from the pressure inside the tube.

12200	**Tube :** Hydraulic model for steady state flow
Nb: 2	
Param1 = C_t (tube longitudinal conductivity)	
Param2 = C_n (wall-through conductivity)	

12210 : Tube : Transient flow

The pressure inside the tube is different from the pressure on its outside wall for the surrounding porous matrix. This model allows calculating transient flow.

Constitutive law: $q = -C_t \nabla P$, $C_M \dfrac{\partial P}{\partial t} = \nabla.(C_t \nabla P) + C_n (p - P)$

P : pressure inside the tube which can be different from the outside pressure
∇P : fluid pressure gradient along the tube line
p : pressure outside the tube

q : debit in the tube, ∇p : fluid pressure gradient along the tube line

Nb = 3
Param1 = C_t (tube longitudinal conductivity)
Param2 = C_n (wall-through conductivity, zero if impervious wall)
Param3 = C_M (storage coefficient)

Note: See the note for 12200.

12210 **Tube :** Hydraulic model for transient flow

Nb = 3
Param1 = C_t (tube longitudinal conductivity)
Param2 = C_n (wall-through conductivity)
Param3 = C_M (storage coefficient)

II.2) Hydraulic - ROCKJOINTS & FRACTURES

See the **General Note** 22210 at the end of this section explaining the parameters of interface model for flow.

22100 : Hydraulic rock joint, *infinite* transverse conductivity

Constitutive law: $q = -C_t \nabla p$

q : debit in the fracture, ∇p : fluid pressure gradient along the fracture line

Nb = 1

Param1 = C_t (tangent conductivity)

Note: Infinite transvers conductivity means that the pressure is the same on the two sides of the fracture or joint element. If the joint is assimilated to a thin layer of thickness e of a porous material with permeability k (see the material 32100), then the equivalent C_t would be $C_t = ke$ and q would represent the integral of velocity in the section (thickness) of the fracture ($q = ve$).

22100	Hydraulic interface with *infinite* transverse conductivity
Nb: 1	
Param1 = C_t (tangent conductivity)	

22110 : Transient hydraulic flow in rock joint, *infinite* transverse conductivity

Constitutive law: $q = -C_t \nabla p \;,\quad C_M \dfrac{\partial p}{\partial t} = \nabla.\left(C_t \nabla p\right)$

q : debit in the fracture, ∇p : fluid pressure gradient along the fracture line

Nb = 2

Param1 = C_t (tangent conductivity)

Param2 = C_M (storage coefficient)

Note: For the infinite transvers conductivity see the note for the material 22100

22110	Hydraulic interface with *infinite* transverse Conductivity, transient flow
Nb: 2	
Param1 = C_t (tangent conductivity)	
Param2 = C_M (storage coefficient)	

22200 : Hydraulic flow in rock joint, *finite* transverse conductivity

Constitutive law: $q = -C_t \nabla P$, $V_n = C_n \llbracket p \rrbracket$

q : debit in the fracture, ∇p : fluid pressure gradient along the fracture line
V_n: The fluid velocity perpendicular to the interface. Its is the average value of the normal fluid velocity in the matrix on the two sides of the joint element.
$\llbracket p \rrbracket$: pressure discontinuity (jump) across the interface

Nb = 2
Param1 = C_t (tangent conductivity)
Param2 = C_n (transverse or normal conductivity)

Note: For this model the pressure is discontinuous across the fracture (pressure jump between the two sides of the fracture). The only case with clear physical meaning is then the case $C_n = 0$ for witch the fracture acts as a barrier to the flow perpendicular to its surface. The variable P in $q = -C_t \nabla P$ represents the mean value of the pressure on the two sides, $(p^+ + p^-)/2$. The case C_t=0 corresponds to an empty joint with no flow through it.

22200	Hydraulic interface with *finite* transverse conductivity
Nb: 2	
Param1 = C_t (tangent conductivity)	
Param2 = C_n (transverse or normal conductivity)	

22210 : Transient hydraulic flow in rock joint, *finite* transverse conductivity

Constitutive law: $q = -C_t \nabla P$, $V_n = C_n \llbracket p \rrbracket$, $C_M \dfrac{\partial p}{\partial t} = \nabla.(C_t \nabla p)$

q : debit in the fracture, ∇P : fluid pressure gradient along the fracture line
V_n: The fluid velocity perpendicular to the interface. Its is the average value of the normal fluid velocity in the matrix on the two sides of the joint element.
$\llbracket p \rrbracket$: pressure discontinuity (jump) across the interface

Nb = 3
Param1 = C_t (tangent conductivity)
Param2 = C_n (transverse or normal conductivity)

Param3 = C_M (storage coefficient)

22210	Hydraulic interface with *finite* transverse conductivity
Nb: 3	
Param1 = C_t (tangent conductivity)	
Param2 = C_n (transverse or normal conductivity)	
Param3 = C_M (storage coefficient)	

General Note 22210: If the joint element represents a thin layer of thickness e constituted of porous material with permeability k and storage coefficient c_m then its tangent and normal conductivities and storage coefficient C_M are given respectively by:

$$C_t = k\, e \ , \quad C_n = k/e \ , \quad C_M = c_m\, e$$

For the permeability k and the storage coefficient c_m of the bulk material see the note for the material 32100.

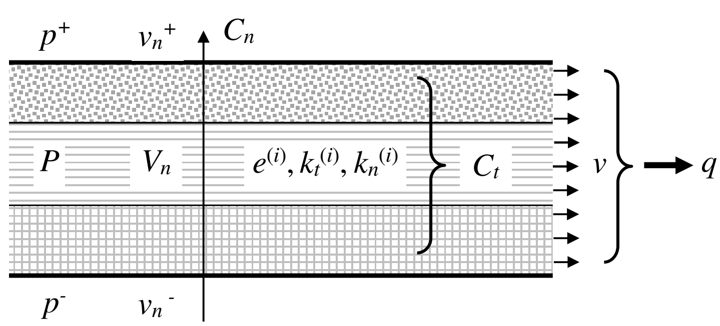
The debit q corresponds to the integral of the fluid velocity on the layer section, and P is the average pressure in the layers:

$$q = \int v\, de \ , \qquad P = \frac{1}{e}\int p\, de$$

If the permeability is high and thickness small, there is continuity of pressure across the joint element (no pressure difference between the two sides). In this case an infinite value of C_n is to be modelled. To avoid numerical problems, in this case a new model is defined in Disroc (models 22100 and 22110) which implicitly supposes the pressure equality on the two sides and does not need a C_n value. On opposite, if $C_n = 0$ then the interface acts as a barrier to the flow perpendicular to its surface.

If the joint element represents an assemblage of several thin layers of bulk materials (three layers in the figure) with which layer (i) having a thickness $e^{(i)}$, a permeability $k_t^{(i)}$ in the direction parallel to the layer and $k_n^{(i)}$ in the direction perpendicular to it and storage coefficient $c_m^{(i)}$, then the equivalent C_t and C_n for the joint element are given by:

$$C_t = \sum_i e^{(i)} k_t^{(i)} \ , \qquad \frac{1}{C_n} = \sum_i \frac{e^{(i)}}{k_n^{(i)}} \ , \qquad C_m = \sum_i e^{(i)} c_m^{(i)}$$



Concerning the flow perpendicular to the fracture, we note that v_n^+ and v_n^- are the flow *in the matrix* perpendicular to the fracture and $V_n = (v_n^+ + v_n^-)/2$.

II.3) Hydraulic - BULK MATERIALS

32100 : Darcy flow with isotropic permeability

Constitutive law: $\underline{v} = -k\, \underline{\nabla p}$

v : fluid velocity in the porous material , $\underline{\nabla p}$: fluid pressure gradient vector

k is a conductivity parameter which is called "permeability" for simplicity. It is related to the intrinsic permeability k_{in} and the Darcian permeability k_{Darcy} by the following relations:

$$k = \frac{k_{in}}{\mu} = \frac{k_{Darcy}}{\rho_f g}$$

Where μ is the dynamic viscosity and ρ_f the specific mass of the fluid and g the gravitational acceleration.

In SI system of units with $\underline{v}$ (m/s), k_{in} (m^2), μ ($Pa.s$), ρ_f (Kg/m^3), g (ms^{-2}) and k_{Darcy} (m/s), the parameter k is expressed in $m^2/(Pa.s)$ or equivalently in $(m/s)/(Pa/m)$.

Note that for water:

$$\mu = 1.01\times10^{-3}\ \text{Pa.s} \qquad \rho_w g = 9.81\times10^{3}\ \text{Pa}/m$$

So, if, for instance, a fluid with the relative density γ is considered (fluid density γ times greater than water) and if the pressure is expressed in MPa, distances in m and the fluid velocity in m/s, then we have $\rho_f g = \gamma\,(9.81\times10^{-3}) MPa/m$. Then the Disroc permeability parameter k ($m^2/MPa.s$) have the following value function of k_{Darcy} (m/s):

$$k = \frac{1}{\gamma}\,\frac{k_{Darcy}}{9.81\times10^{-3}}$$

Nb = 1
Param1 = k (permeability)

32100	Darcy's law with isotropic permeability
Nb: 1 Param1 = k (permeability)	

32110 : Transient Darcy flow with isotropic permeability

Constitutive law: $\underline{v} = -k\,\underline{\nabla p}$, $C_M \dfrac{\partial p}{\partial t} = div(k\nabla p)$

v : fluid velocity in the porous material , $\underline{\nabla p}$: fluid pressure gradient vector

For the definition of the unit of k see the material 32100.
C_M has the dimension and the unit of the pressure p.

Nb = 2
Param1 = k (permeability)
Param2 = C_M (storage coefficient)

32110	Transient Darcy flow with isotropic permeability
Nb: 2	
Param1 = k (permeability)	
Param2 = C_M (storage coefficient)	

32111 : Transient Darcy flow with evolving permeability (GeliSol)

Constitutive law: $\underline{v} = -\dfrac{k_{Darcy}}{\gamma_w} k_r(S_\lambda)\,\nabla(p+\gamma_w z)$, $C_M \dfrac{\partial p}{\partial t} = div\,\underline{v}$

$$k_r(S_\lambda) = \sqrt{S_\lambda}\left(1-(1-S_\lambda^{1/m})^m\right)^2$$

v : fluid velocity in the porous material ,
p : fluid pressure, $\nabla(.)$: gradient vector
γ_w : fluid (water) unit weight (= $\rho_w g$),
C_M : Storage coefficient (= $1/M$ with M the Biot Modulus for poroelastic material)
k_{Darcy} : Darcy's permeability
k_r : relative permeability
m : positive constant parameter
S_λ : Degree of saturation

Note 210519
S_λ is calculated from the relation S_λ= 1- V^{in}_h where V^{in}_h is an internal variable which can be given by the user in the User module in the array Vinh(n,1). For the material GeliSol, it is automatically calculated from the freezing curve of the material which provides the degree of saturation in liquid water, S_λ function of the temperature.

Nb = 4
Param1 = k_{Darcy} (permeability)

Param2 = C_M (storage coefficient)
Param3 = γ_w (water unit weight)
Param4 = m

Internal variable:
$V^{in}_h(n,1)$: internal variable $1-S_\lambda$

32111	Transient Darcy flow with evolving Permeability (Gelisol)

Nb: 4
Param1 = k_{Darcy} (permeability)
Param2 = C_M (storage coefficient)
Param3 = γ_w (water unit weight)
Param4 = m

32200 : Darcy flow with anisotropic permeability

Constitutive law: $\underline{v} = -\boldsymbol{k}\ \underline{\nabla p}$,
v : fluid velocity in the porous material , $\underline{\nabla p}$: fluid pressure gradient vector

For the definition of the unit of k see the material 32100.

Nb = 3
Param1 = k_{xx}
Param2 = k_{yy}
Param3 = $k_{xy} = k_{yx}$

32200	Darcy's law with anisotropic permeability

Nb: 3
Param1 = k_{xx}
Param2 = k_{yy}
Param3 = $k_{xy} = k_{yx}$

32210 : Transient Darcy flow with anisotropic permeability

Constitutive law: $\underline{v} = -\boldsymbol{k}\ \underline{\nabla}p$, $C_M \dfrac{\partial p}{\partial t} = div(\boldsymbol{k}\nabla p)$

v : fluid velocity in the porous material , $\underline{\nabla}p$: fluid pressure gradient vector

For the definition of the unit of k see the material 32100.
C_M has the dimension and the unit of the pressure p.

Nb = 4
Param1 = k_{xx}
Param2 = k_{yy}
Param3 = $k_{xy} = k_{yx}$
Param4 = C_M (storage coefficient)

32210	Transient Darcy's law with anisotropic permeability
Nb: 4 Param1 = k_{xx} Param2 = k_{yy} Param3 = $k_{xy} = k_{yx}$ Param4 = C_M (storage coefficient)	

II.4) Hydraulic 40000 Cables
→ See 12200, 12210 Tubes

II.5) Hydraulic 50000 Beams
→ See 12100, 12110 Boreholes

II.4) Hydraulic 60000 Bolts
→ See 12200, 12210 Tubes

III) Thermal

III.1) Thermal - WIRES & TUBES
(associated thermal model for bars, beams, anchors and bolts)
III.2) Thermal - ROCKJOINTS & FRACTURES
III.3) Thermal - BULK MATERIALS

33111 : Transient Heat flow with thawing (GeliSol)

Constitutive law of the material includes the equations of heat transport by thermal diffusion (Fourier's law) and by advection. In the interval of temperatures corresponding to the thawing process, the liquid water content decreases because the water is transformed into ice (see the figure). In these temperatures interval, the thermal capacity includes the latent heat of the water to ice phase change *L*.

$$\underline{J}_D = -\Lambda\,.\nabla T \;\;,\qquad \underline{J}_A = \rho_\lambda C_\lambda^p T\,\underline{v}\;,\qquad \underline{J} = \underline{J}_D + \underline{J}_A \tag{3.1}$$

$$\left(\rho C^p + \rho_\lambda L\, G\, \phi\right)\frac{\partial T}{\partial t} = div(\Lambda\,.\nabla T) - div\left(\rho_\lambda C_\lambda^p T\,\underline{v}\right) \tag{3.2}$$

$$G(T) = \frac{\partial S_\lambda(T)}{\partial T} \tag{3.3}$$

T : temperature,
∇T : temperature gradient,
$\underline{J}_D$: diffusive heat flow,
$\underline{J}_A$: advective heat flow,
Λ : thermal conductivity,
ρ : mass density (of the porous material, soil or rock),
C^p : specific heat capacity of the porous material (soil, rock) at constant pressure,
ρ_λ : pore fluid (water) mass density,
$C^p{}_\lambda$: pore fluid (water) specific heat capacity,
ϕ : porosity,
L : latent heat of the ice–water phase change (heat needed for unit mass change),
$S_\lambda(T)$: liquid saturation degree at temperature T

New variables are defined for simplicity:

- L_v : volumetric latent heat of the water-ice phase change. $L_v = \rho_\lambda L$ where ρ_λ is the water density and L the (specific) latent heat of the ice–water phase change
- $C_{v\lambda}$: volumetric heat capacity of the liquid (water) $C_{v\lambda} = \rho_\lambda C^p{}_\lambda$ where ρ_λ is the density and $C^p{}_\lambda$ the specific heat capacity at constant pressure of the liquid.
- C_{vu} : volumetric heat capacity of the unfrozen soil: $C_{vu} = \rho C^p$ where ρ is the density and C^p the specific heat capacity at constant pressure of the soil at unfrozen state,

C_{vf} : volumetric heat capacity of the frozen soil: $C_v = \rho C^p$ where ρ is the density and C^p the specific heat capacity at constant pressure of the soil at frozen state,
C_{vs} : volumetric heat capacity of the partially frozen soil:

$$C_{vs} = S_\lambda\, C_{vu} + (1\text{-}S_\lambda)\, C_{vf}$$

The heat conductivity varies also with the water content between the values corresponding to the unfrozen and frozen states:

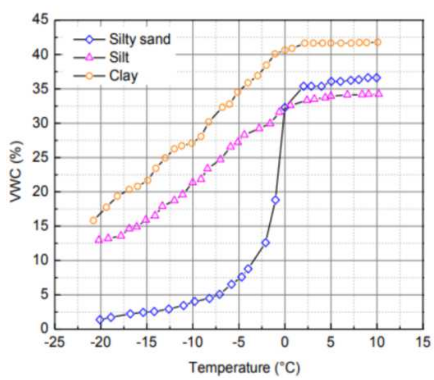
$$\Lambda = S_\lambda\, \Lambda_u + (1\text{-}S_\lambda)\, \Lambda_f$$

Λ_u : heat conductivity of the soil at unfrozen state,
Λ_f : heat conductivity of the soil at frozen state.

With these notations the equation (3.2) reads:

$$\left(C_{vs} + L_v\; G\; \phi\right)\frac{\partial T}{\partial t} = div(\Lambda\, .\nabla T) - div\left(C_{v\lambda} T \underline{v}\right) \tag{3.4}$$

The evolution of S_λ with temperature is deduced from the freezing curve of the soil, the material data giving the evolution of the liquid water content in the soil at different temperatures:



W_c : unfrozen water content of the partially frozen soil
W_c^{Max} : water content of the unfrozen soil

$$S_\lambda(T) = \frac{W_c(T)}{W_c^{Max}}$$

$$G(T) = \frac{\partial S_\lambda(T)}{\partial T}$$

Variation of the water content with the temperature for different soils (Li *et* al., 2018)
Li, H., Yang, Z. J., and Wang, J. (2018). Unfrozen water content of permafrost during thawing by the capacitance technique. *Cold Regions Science and Technology*, 152 :15-22.

Different options exist to define and introduce the function $S_\lambda(T)$ in the model:

- If the parameter *Option* = 0 there is no thawing modeled.
- If the parameter *Option* = 1 then a simple model of thawing is considered: $S_\lambda(T)$ varies linearly between T=T_{min}<0 and T=0 from $S_\lambda(T_{min})$= S_{min} to $S_\lambda(0)$=1. In this case T_{min} and S_{min} are given as parameter of the material (Param7, Param8).
- If the parameter *Option* = 2, the thawing curve is defined in a file. The file is text file called *name*.dat where *name* is the name of the material. This file has the following format:

#Comments: the thawing curve for the material "Clay"
#The curve includes *N* points
Curve
N
T_1 S_1

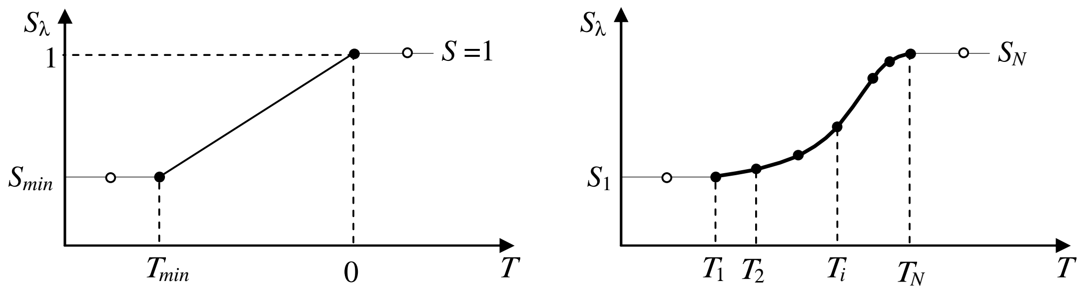
T_2 S_2
…
T_N S_N

____

The lines before the line starting by keyword 'Curve' are free comments. The line containing just the keyword 'Curve' is mandatory. Then follows N, the number of points, and then N lines containing the pair 'T_i S_i' where T_i are increasing temperatures and S_i the liquid water contents with values between 0 and 1. Then the function $S_\lambda(T)$ is built in the following way:

$S_\lambda(T)=S_1$ if $T \le T_1$,
S_λ varies linearly from S_i to S_{i+1} for $T_i \le T \le T_{i+1}$,
$S_\lambda(T)= S_N$ if $T_N \le T$

Un example of thawing curve file is given in the folder Tools, called thawing.dat.



Option 1: Curve defined by (T_{min}, S_{min}) *Option* 2: Curve defined by (T_i, S_i) in a file

Figure : Two options of definition of the thawing curve

Note 210521:
The thawing curve $S_\lambda(T)$ is a characteristic of the soil and an input of the model. From this data, at each temperature, the S_λ is determined. This value is used by the hydraulic and mechanical material models 32111 and 31121 (Gelisol) in order to express the effects of thawing process on the hydraulic (permeability) and mechanical properties.

Nb = 10
Param1 = Λ_u : thermal conductivity of the unfrozen soil,
Param2 = Λ_f : thermal conductivity of the frozen soil,
Param3 = C_{vu} : volumetric heat capacity of the unfrozen soil,
Param4 = C_{vf} : volumetric heat capacity of the frozen soil,
Param5 = $C_{v\lambda}$: volumetric pore fluid (water) heat capacity,
Param6 = L_v : volumetric latent heat of the water-ice phase change,
Param7 = ϕ : porosity,
Param8 = *Thawing Option* for the definition of the thawing curve $S_\lambda(T)$,
Param9= T_{min}
Param10= S_{min}

Internal variable:
$V^{in}_T(n,1)$: internal variable $1-S_\lambda(T)$

33111 Transient heat flow with thawing (Gelisol)

Nb = 10
Param1 = Λ_u : unfrozen soil thermal conductivity,
Param2 = Λ_f : frozen soil thermal conductivity,
Param3 = C_{vu} : unfrozen soil volumetric heat capacity,
Param4 = C_{vf} : frozen soil volumetric heat capacity,
Param5 = $C_{v\lambda}$: pore fluid (water) volumetric heat capacity,
Param6 =L_v :water-ice phase change volumetric latent heat,
Param7 = ϕ : porosity,
Param8 = *Thawing Option* (0,1,2)
Param9= T_{min}
Param10= S_{min}

IV) Custom Special Models

HiDCon : High Deformable Concrete

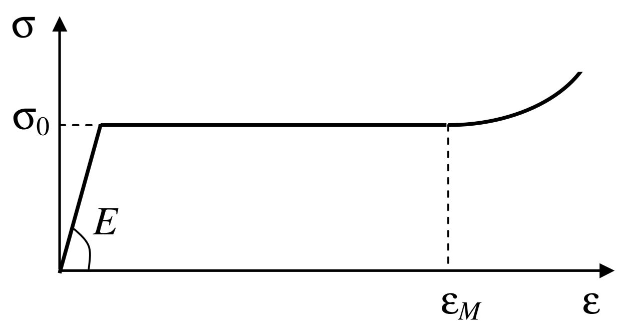
The elastic-plastic behavior of Highly Deformable Concrete Elements (HiDCon) can be modeled by a plane Mohr-Coulomb criterion with hardening. With σ_{zz} supposed to be the intermediate principal stress and with negative compression sign convention the Mohr-Coulomb criterion reads:

$$F(\boldsymbol{\sigma},\xi) = (\sigma_1 - \sigma_2) + (\sigma_1 + \sigma_2)\sin\phi - 2C(\xi)\cos\phi \ \le\ 0$$

Where ξ is the hardening parameter and (σ_1, σ_2) the major and minor in-plane principal stresses. The compressive strength R_c (the UCS) is related to the cohesion by:

$$R_c(\xi) = \frac{2C(\xi)\cos\phi}{1-\sin\phi}$$

And it varies with the plastic strain according to the hardening rule. This rule is chosen in a way to have the typical behavior of HiDCon elements illustrated in the following figure.



Under a uniaxial stress, the deformation is elastic and linear up to the stress σ_0 for the axial strain $\varepsilon_0 = \sigma_0/E$ and then a perfect plastic strain takes place up to a total axial strain ε_M. After this stage, the stress increases with a paste which is a quadratic function of the plastic strain.

Constitutive model:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

Elasticity:

$$\dot{\boldsymbol{\varepsilon}}^e = \frac{1+\nu}{E}\dot{\boldsymbol{\sigma}} - \frac{\nu}{E} tr(\dot{\boldsymbol{\sigma}})\,\boldsymbol{\delta}\,,$$

Plasticity:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\frac{\partial G}{\partial \boldsymbol{\sigma}}\ , \quad \begin{array}{l} if\ F<0\ then\ \ \dot{\lambda}=0 \\ if\ F=0\ then\ \ \dot{\lambda}\ge 0,\ \dot{F}\le 0,\ \dot{\lambda}\dot{F}=0 \end{array}$$

Plastic potential:

$$G(\boldsymbol{\sigma}) = (\sigma_1 - \sigma_2) + (\sigma_1 + \sigma_2)\sin\psi$$

Hardening rule:

$$R_c(\xi) = \sigma_0 + \beta E\left\langle \xi - \varepsilon_0^p \right\rangle^2 \qquad \dot{\xi} = \alpha\sqrt{\dot{\boldsymbol{\varepsilon}}^p : \dot{\boldsymbol{\varepsilon}}^p}$$

where the hardening variable ξ starts from 0 at the initial state of the material, the symbol $\langle\ .\ \rangle$ stands for the positive part:

$$< x > = 0 \quad if \quad x < 0$$
$$< x > = x \quad if \quad x \geq 0$$

and $\varepsilon_0^p = \varepsilon_M - \dfrac{\sigma_0}{E}$ and β a material parameter. α is an internal constant parameter ensuring that for a unixial compression test ξ represents the axial plastic strain.

Nb = 7
Param1 = E
Param2 = ν
Param3 = σ_0 (initial UCS)
Param4 = ϕ (°)
Param5 = ψ (°)
Param6 = ε_M
Param7 = β

Internal variable
Vin(n,1) : ξ
Necessary Condition on parameters: $\varepsilon_M > \sigma_0/E$